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Math. 1113

A
Familiar Introduction
TO THE
THEORY and PRACTICE
OF
PERSPECTIVE.

By JOSEPH PRIESTLEY, LL.D. F.R.S.

ΚΑΙ ΑΓΕΩΜΕΤΡΗΤΟΣ ΕΙΣΙΤΩ.

LONDON:
Printed for J. Johnson and J. Payne, at
N° 8. Pater-noſter-Row.
M.DCC.LXX.

TO

SIR JOSHUA REYNOLDS, KNT. F.R.S.

PRESIDENT

OF THE

ROYAL ACADEMY OF PAINTING, &c.

THIS TREATISE ON PERSPECTIVE

IS, WITH GREAT RESPECT,

INSCRIBED BY

HIS MOST OBEDIENT

HUMBLE SERVANT,

JOSEPH PRIESTLEY.

LEEDS,
MARCH 20, 1770.

THE PREFACE.

THE art of drawing in perſpective has ſo many, and ſuch obvious uſes, that there can be no occaſion to enumerate them. Thoſe who want to communicate their ideas to others often feel the inperfection of *ſounds*, or of characters that only repreſent ſounds, for this purpoſe; and thoſe who are deſirous of receiving information find, that it is, in many caſes, conveyed with unſpeakably more eaſe and certainty, by the eye, than by the ear.

The reaſon is, that every *verbal* deſcription of any object that hath the dimenſions of length, &c. muſt be reduced to a *lineal* one in the imagination, before any diſtinct idea can be conceived of it; and this is often a difficult and painful operation.

It is particularly fatiguing to the imagination to put together the parts of a complex object, when the deſcription of each of them is made ſeparately; for each part being conceived very imperfectly, and one part being in danger of being forgotten, while the mind is engaged in attending to another, the conception of the whole cannot but be very obſcure, and inadequate. The ſymmetry and elegance of a pile of building is abſolutely loſt without a drawing, and the moſt maſterly painters are incapable of producing any thing but the moſt hideous and unnatural groupes of figures, without the aſſiſtance of this art.

Of all the *imitative arts*, this of perſpective is capable of being brought, and indeed has actually been brought the neareſt to perfection; becauſe it is wholly within the ſphere of *mathematical ſcience*, a branch of knowledge which has been moſt diligently cultivated of late years, and with reſpect to the objects of which, we are not uſed to acquieſce in any thing ſhort of abſolute certainty. Accordingly, we ſee, that there is no object of ſight, be it ever ſo complex, which thoſe perſons who have applied themſelves to the ſtudy of this art are not able to repreſent, juſt as it appears to the eye. By this means the ideas of all the beauties of nature and art are faithfully preſerved, and thoſe perſons who have no opportunity of ſeeing the objects themſelves, may ſtudy, and be delighted with them, in the works of travellers and natural hiſtorians; and philoſophers can deſcribe their machines, and their apparatus for making experiments, without any danger of embarraſſing themſelves, or of perplexing and confounding their readers.

As in all the other branches of mathematical knowledge, the progreſs of this art has been ſlow, but ſure; and the Engliſh writers (particularly Dr. Brooke Taylor) ſeem to have carried it to a degree of perfection we can hardly conceive it poſſible to be exceeded. As much, however, is known, as we can imagine will ever be made uſe of.

All that is wanting ſeems to be a method of facilitating the attainment of this art; and in this reſpect I cannot help thinking there is room to improve upon all the books that I have yet ſeen upon this ſubject. Indeed the actual ſtate of the practice of this art ſeems to be a proof that the attainment of it has not been ſufficiently eaſy. For of the great numbers, both of ladies and gentlemen, who learn to draw, and even take pains to draw with elegance, there are very few who ſo much as attempt to learn perſpective. Though, therefore, they be able to copy prints, and the deſigns of others, and to execute theſe drawings in a maſterly manner, they are utterly incapable of *deſigning* any thing themſelves, or of drawing from nature; ſo that a perſon who can even draw human figures, in every variety of attitude and paſſion, ſhall not be able to take a correct drawing of a table, a chair, or the moſt ſimple machine, or inſtrument, that requires nothing but ſtraight lines.

Of thoſe gentlemen who travel, or go long voyages, how few are furniſhed with the rudiments of this art, for the exerciſe of which they have perpetual occaſion, if they would preſerve the ideas of the memorable ſcenes and objects they meet with, or communicate any juſt notion of them to their friends. How many philoſophers, and even perſons who have been no mean proficients in other branches, both of ſpeculative and practical mathematics, do we hear complaining of their ignorance of this art, and of the difficulties they have met with in their attempts to learn it; ſo that they are incapable of making a drawing of the apparatus they make uſe of in their experiments, and are always obliged to employ a profeſſed artiſt for this purpoſe. Theſe difficulties have diſcouraged numbers, who have not ſcrupled to give the very great price that books of perſpective generally ſell for, and who have been willing to take a good deal of pains in the ſtudy of them.

Though, for my own part, I got a general idea of the theory of perſpective pretty early, at the time that I attended to other branches of mathematical ſcience, I was not capable of making a draught of any thing, till I was under a neceſſity of having original drawings of electrical machines and apparatus, and was in a ſituation where I could not find any perſon to make them for me. At firſt I puzzled myſelf with ſeveral mechanical methods of drawing; but though I made conſiderable improvements in ſome of them, I was obliged, at laſt, to have recourſe to the rules of perſpective. I found them, however, ſo immethodically digeſted, or ſo inſufficiently explained, that, in ſeveral caſes, I was able to inveſtigate the rule myſelf, from conſidering the nature of the thing, ſooner than I could find it in the books; and after all, the drawings that I did make at that time were executed when I had a very imperfect knowledge of the art.

The embarraſſment I then found myſelf in, made me attend to the ſubject afterwards,

wards, when I was more at leiſure for it. Having ſtruggled with the difficulties myſelf, and writing while the idea of them is freſh in my memory, I hope that I have been better able to obviate, or remove them, for the benefit of others. I have been willing, however, to make the attempt; and I flatter myſelf that any perſon, of the age and qualifications of thoſe who ever think of learning to draw, may, by the help of this treatiſe, without any inſtructor, make themſelves maſters of every thing that is eſſential to this art. Leſs than a week, I am pretty clearly of opinion, would be ſufficient for a maſter of the art to inſtruct another in it, in the method here laid down; and a few hours would be ſufficient to give a perſon, who has a previous knowledge of geometry, a perfect idea of all the real varieties that can poſſibly occur in the practice of it.

Nor will this be any matter of wonder when it is conſidered, that after the preparation of the drawing-board, the whole of this art conſiſts in drawing the perſpective appearance of lines in no more than *five different varieties of poſition*, and in fixing points at given diſtances in thoſe lines.

In the method deſcribed, Part IX, nothing is requiſite, but to take the elevation and declination of any *ſingle point* in an object, ſo that the whole myſtery of it might be learned in a few minutes. I therefore cannot help wiſhing that mathematical inſtrument-makers would conſtruct ſome cheap inſtrument for this purpoſe. The contrivance would be very eaſy, and it would greatly facilitate the practice of this uſeful art. Whether this method be explained in any other treatiſe I cannot tell. It occurred to me from conſidering the nature of the thing; I practiſed it before I was acquainted with any other; and the method, both with reſpect to theory and practice, is intirely independent of the common method, and abundantly more ſimple. The common perſpective is founded on the conſideration of *planes* and *lines*, but in this *points* only, in which they terminate, are regarded. But it is better adapted to the purpoſe of drawing large objects, in the open air, than ſmall objects within doors.

The reader will obſerve that, beſides the rules of *orthographical* perſpective, laid down in Part X, this treatiſe contains a deſcription of three diſtinct methods of drawing in common or *ſcenographic* perſpective, each of which may be learned independent of the others.

That which is of the moſt general application is contained in Parts III, IV and V; and a ſummary view of it is given in Part VI. By this method a perſon will be able to draw the perſpective appearance of all objects whatever, their dimenſions and diſtances being previouſly known. There is no occaſion for their being in view, or for their ichnography being taken.

The ſecond method, contained in Part VIII, teaches how to draw the appearance of objects from their ichnography, or ground plan. The rules of it are exceeding few and eaſy, but the true dimenſions of things muſt be taken before their ichnography can be drawn. If any perſon would chuſe to confine himſelf to this method;

thod; after learning to draw the appearance of objects on the ground plane, he muſt conſult Part V. Sect. II. Caſe I. for a method of raiſing perpendicular altitudes upon any part of that plane; and then the method will be complete.

The third method is that mentioned above, which ſuppoſes the objects to be in view, and by finding the ſituation of points, determines the appearance of objects. This muſt be done by an inſtrument, but one ſingle rule comprehends the whole practice.

Mathematicians will perhaps be ſurpriſed to find ſo little *theory* in this treatiſe, but of this I have made no parade, contenting myſelf with giving a ſatisfactory reaſon for every eſſential part of the practice; and if theſe ſatisfactory reaſons be given, it is all that the reader can reaſonably require. Dr. Brooke Taylor, and other geometricians, appear to me to have made the theory of perſpective much more extenſive and complex, than the *practice* requires; and to have introduced more *technical terms* than are really neceſſary for this purpoſe. Conſidering perſpective as a branch of geometry, I am far from blaming the conduct of thoſe great writers; but I would not have the young artiſt be diſcouraged, by imagining that he muſt neceſſarily make himſelf maſter of their works, in order either to practiſe this art, or to be fully convinced of the reaſons on which it is founded.

The reader is indebted for part of this work to Mr. Joſeph Prieſtley, of Halifax, from whoſe knowledge of mathematics in general, and of this branch in particular, I once expected a much more complete and elegant view of the theory of this art; and I do not yet deſpair of his undertaking ſo uſeful a work. He drew up the *general view of the theory of perſpective*, prefixed to the Notes, and wrote all thoſe paragraphs in which the propoſitions in that piece are referred to. He gave me the method of meaſuring a line oblique to the ground plane, deſcribed p. 34, with the demonſtration of it, and the method of drawing circles deſcribed Caſe I and II. Part VII. He was alſo ſo obliging as to aſſiſt me in reviſing and correcting the whole work.

Let it be obſerved, that I term this treatiſe only a *familiar introduction* to the theory and practice of perſpective. It is therefore by no means intended to ſuperſede other valuable works, that contain a greater variety of examples, and a detail of particular proceſſes, which are highly uſeful to thoſe who have much practice in this art. I flatter myſelf that by the help of this *introduction*, thoſe books will be much better underſtood, and more uſeful than ever.

As a friend to the *arts*, but more eſpecially to *ſcience*, which is greatly indebted to this particular art, I ſhall think myſelf happy, if, either by what I have written myſelf, or by recommending the writings of others, I ſhall contribute to make the buſineſs of perſpective more generally underſtood and practiſed. Every boy that is capable of being taught writing and accompts, might be perfectly inſtructed in all the rules contained in Parts III, IV, and V, of this Introduction, in a few weeks. They take up no more than 24 pages of the work, not conciſely written. And every ſchoolmaſter, who is as yet unacquainted with the rudiments of this uſeful art, might make himſelf maſter of them in a few evenings.

There is no occaſion to trouble every boy with the *theory* of perſpective; but I would have all young perſons, without exception, made ready in the *practice*. Thoſe who apply to any branch of the mathematics, may learn the theory afterwards. And if a perſon were taught no more than the *neceſſary rules* in his youth, he might learn *particular improvements* in drawing at his leiſure, whenever he ſhould have occaſion for them.

Since this Work was printed off, I have ſeen a ſubſtance excellently adapted to the purpoſe of wiping from paper the marks of a black-lead-pencil. It muſt, therefore, be of ſingular uſe to thoſe who practiſe drawing. It is ſold by Mr. NAIRNE, *Mathematical Inſtrument-Maker, oppoſite the Royal-Exchange. He ſells a cubical piece, of about half an inch, for three ſhillings; and he ſays it will laſt ſeveral years.*

A FAMILIAR INTRODUCTION To the THEORY and PRACTICE of PERSPECTIVE.

PART I.

Of the Inſtruments that are of Uſe in the Practice of PERSPECTIVE, *and the Application of them.*

A Perſon who propoſes to practice the art of drawing in perſpective muſt provide himſelf with a *drawing board and ſquare*, and a *caſe of mathematical inſtruments*, conſiſting of a pair of compaſſes, a ſet of ſcales, a ſector, a protractor, and a drawing pen.

The *drawing board* is made in the form of a ſquare, or parallellogram; and if any one of the angles be a right angle, it will be ſufficient, provided the ſquare which is uſed in drawing lines upon it be always applied to one of the ſides that contain that right angle; and lines may be drawn in all poſſible directions without uſing any other ſide of the board. It will be very convenient for a perſon who practices drawing much, to have ſeveral drawing boards, of different ſizes, for the ſake of having greater variety in the compaſs of his draughts.

The *ſquare* to be uſed along with the drawing board is a flat ruler, as *a*, Fig. 1, at one end of which are faſtened two tranſverſe pieces; one of them, *b*, fixed at right angles to it, and the other *c*, moveable, ſo as to be fixed at any angle required. Theſe tranſverſe pieces are applied cloſe to the ſide of the drawing board, while the ruler lies upon it; and by ſliding them along the board, lines may be drawn parallel to one another with much leſs trouble than by the help of a parallel ruler. By the fixed tranſverſe, lines may be drawn parallel to one ſide of the board and perpendicular to the other; and by the moveable tranſverſe, lines that have any degree of obliquity to the ſides of the board may be drawn parallel to one another; and if, without moving the tranſverſe, the ruler be removed to the other ſide of the board, lines may be drawn perpendicular to them. But if the obliquity be very great, it will be impoſſible to apply the ſquare, ſo as to interſect the lines at right angles in ſome parts of the board. In this caſe, recourſe muſt be had to a *parallel ruler*.

A great variety of *ſcales* is uſeful, in order to lay down lines of any given length, in whatever proportion is moſt convenient, with reſpect to the ſize of the drawing, &c. If none of the ſcales happen to ſuit the purpoſe, recourſe muſt be had to the *line of lines* upon the ſector: for, by the different openings of that inſtrument, a line of any length may be divided into as many equal parts as a perſon chuſes.

The only lines that are neceſſary to be attended to upon the ſector, beſides theſe, for the purpoſe of drawing, are the two *lines of tangents*; one of them marked T, beginning at the center of the inſtrument, and ending with 45 at the extremity of the leg; the other marked *t*, beginning at the diſtance of about one-fourth of the radius from the center, and reaching a little beyond 75, near the extremity.

Leſt a book explaining the uſe of the ſector ſhould not happen to be at hand, I ſhall juſt inform the perſon who is learning to draw, that, if he want the tangent of an angle exceeding 45 degrees, he muſt open the ſector till the 45 on the lines marked *t* be ſet at the ſame diſtance, at which the two forty-fives on the lines marked T were placed; and then take the tangent required, juſt as he would have done, if the tangent had been leſs than 45, and it had not been neceſſary to make any other opening of the inſtrument.

If the ſector be ſo conſtructed, as that the 45 *t* begin exactly at the diſtance of one-fourth of the radius from the center, and conſequently the tangent of 45 *t*, be exactly one-fourth of 45 T; there will be no occaſion for making any other opening of the ſector; for the tangent of any degree exceeding 45 being taken on *t*, and repeated four times, will be the length of the tangent required.

Beſides the ſmall ſector in one of the common pocket caſes of inſtruments, I would adviſe a perſon who propoſes to learn to draw to get another, of one foot radius, at leaſt. Two ſectors are, in many caſes, exceedingly uſeful, if not abſolutely neceſſary; and I would not adviſe a perſon to be ſparing of expence in procuring a very good inſtrument, the uſes of which are ſo various and important.

The *protractor* is generally a circle, or ſemi-circle, the limb of which is divided into 360 parts, called degrees. Sometimes, however, a parallellogram, about the ſize of a common ſcale, is uſed as a protractor. In this caſe the edges of it are divided in the ſame manner as if it had been part of a circular piece, and the lines drawn from the center to the circumference; but it is eaſier to lay down any angle with accuracy from a circular, or ſemi-circular protractor.

A *drawing pen* is neceſſary, becauſe a common open pen can hardly be applied to the edge of a ruler without blotting the paper when it is taken up; beſides, by the help of a drawing pen, lines may be made of preciſely the ſame thickneſs throughout, and of whatever thickneſs a perſon pleaſes.

Black lead pencils are very uſeful, in order to draw lines that are of no ſervice, but as a direction to draw other lines by them; becauſe, when they have anſwered this purpoſe, they may be taken out with a few crumbs of ſoft bread.

PART II.

The Definition of neceſſary technical Terms, and the Preparation of the Drawing-Board.

PERSPECTIVE is the art of delineating objects as they would appear upon an upright plane, interpoſed between them and the eye; for inſtance, as they appear upon a pane of glaſs when they are ſeen through a window. In this manner it is evident, that the images of objects of the ſame ſize will occupy more or leſs ſpace, according as they are nearer or farther off; and the great difficulty of drawing a number of objects conſiſts in giving them theſe proportions. By this art, therefore, the pictures of objects are made to exhibit the ſame appearance upon one plane, and at the ſame diſtance from the eye, that they have in nature upon different planes, and at different diſtances; which is a ſpecies of imitation that is particularly pleaſing to all perſons.

To make the practice of this agreeable art the more intelligible, I ſhall ſuppoſe that I am about to make the perſpective drawing of a number of objects, as *a*, *b*, Fig. 2. which require every variety in the practice; and ſhall minutely deſcribe every part of the proceſs I make uſe of for this purpoſe. [A]

The firſt thing I do is to faſten a ſheet of paper upon my drawing-board, by bits of wafer or ſealing wax, at each corner, in order to make it lie flat and ſteady. The ſurface of this paper is deſigned to repreſent the plane on which the objects are to be drawn, and which is generally called the *perſpective plane.* In other words, it repreſents the glaſs window, which I before ſuppoſed to intervene between my eye and the objects, and upon which their appearance was to be drawn. This appearance I am now to copy on my paper.

In order to this, I draw, or ſuppoſe to be drawn, upon the plane on which the objects ſtand, called the *ground plane,* a line AB, Fig. 2, on which the perſpective plane, through which I view the objects, is ſuppoſed to ſtand. I then take my drawing-board, and, by the fixed tranſverſe of the ſquare, draw the line AB, Fig. 3, to repreſent it. This line I, therefore, call the *ground line* of my piece.

After this I note the point C, Fig. 2, in that part of this ground line which is neareſt to my eye; and upon it I raiſe, or ſuppoſe to be raiſed, the line CE, perpendicular to the other. Alſo, upon my paper, Fig. 3, I draw the line CE, to repreſent it.

The next thing I do is to meaſure the height of my eye above the plane on which the objects ſtand; and ſuppoſing it to be ſix feet, I ſet off from C to D, Fig. 3, the length of ſix diviſions, from any ſcale that I think proper, and draw the line FG parallel to the ground line AB. This line, being drawn at the diſtance of the height of my eye, will repreſent a plane paſſing through my eye, and ſeen edgeways; and being parallel to the horizon (ſuppoſing that I ſtand upright) it is called the *horizontal line*. The point D in this line, to which my eye is directly oppoſite, is called the *point of ſight*. It is that point in the perſpective plane which is the neareſt to my eye.

When I have thus drawn upon my paper the horizontal line, I meaſure the diſtance at which I ſtand from the imaginary perſpective plane; and, ſuppoſing it to be nine feet, I mark a point E, Fig. 3, in the perpendicular line, at the diſtance of nine diviſions from D in the horizontal line. I alſo ſet off the ſame diſtance both ways from the point of ſight D, along the horizontal line to F and G.

This being done, my board and paper are prepared for the delineation of the objects I propoſe to draw. Or, if I pleaſe, I may draw the lines HIKL, to bound the picture; but this may as well be omitted, till the drawing be compleated.

N. B. In all that follows, I ſhall ſuppoſe the drawing-board to be prepared in this manner; at leaſt, that as many of theſe lines are drawn, and as many of the points fixed, as the purpoſe of the picture requires. I ſhall, therefore, hereafter omit the repetition of this part of the proceſs. Let it be obſerved, alſo, that the letters A, B, C, D, E, F, and G, always keep the ſame places, and have the ſame uſes in all the figures belonging to this work.

A general idea of the nature of perſpective, and of the preparation of the drawing-board may, perhaps, be more clearly underſtood by means of Fig. 4. In this figure the plane X, which muſt be laid flat upon the table, is the *ground plane* on which the objects ſtand; the plane Y, which muſt be raiſed perpendicular to the other, is the *perſpective plane* upon which they are to be delineated, and the part Z, which muſt alſo be raiſed perpendicular to the table, repreſents the ſpectator, the line *ab* being the height of his eye above the ground plane. Alſo *b*C is the diſtance at which he ſtands from the perſpective plane Y, on which the ſame lines are drawn as in Fig. 3, beſides others which will be explained in their proper place. If now the plane Y be ſuppoſed to be tranſparent, ſo that objects ſituated on the plane X could be ſeen through it, their appearance on this plane Y would be the perſpective repreſentation required.

* * *

PART

PART III.

To find the perſpective ſituation of right lines upon the ground plane.

THE drawing board being prepared as has been already deſcribed, I proceed to draw upon it the perſpective appearance of all the objects that are ſituated beyond the perſpective plane.

Since the figures of all objects are contained under lines, bounded by points, *i. e.* under lines terminating in different places, it is evident, that the whole art of perſpective conſiſts of nothing more than a method of finding the ſituation of all lines upon the perſpective plane, and of cutting thoſe lines in any given proportion, according to the lengths required. Curves are no exception to this obſervation, ſince they are drawn by fixing a ſufficient number of points in right lines, and joining them by a ſteady hand. In geometry, they are conſidered as conſiſting of an infinite number of right lines. I ſhall, therefore, in the firſt place, explain the method of finding the perſpective ſituation of right lines running in all poſſible directions, beginning with thoſe which are drawn upon the ground plane.

All right lines drawn on the ground plane are either parallel to the ground line, as *cd*, Fig 2, perpendicular to it, as *di*, or oblique to it, as *ml*.

SECTION I.

To find the perſpective ſituation of right lines parallel *to the ground line.*

IF the lines to be drawn in perſpective be parallel to the ground line, as *cd*, Fig 2, they will be parallel to it on the perſpective plane, and conſequently parallel to one another. [B]

Being ſenſible, then, that the image of the line H I, upon the ground plane X, Fig. 4, muſt be drawn parallel to the ground line A B, I only want to know at what diſtance it muſt be drawn from it.

To aſcertain this, I meaſure the perpendicular diſtance CM of the given line from the perſpective plane; and finding it, for inſtance, to be three diviſions, I ſet off that diſtance from C to *g*, and draw the line G*q*, cutting the line CD in *m*; and the line *hi* drawn through *m*; parallel to the ground line, A B is the true perſpective ſituation of the line HI required. [C]

If I would draw another line, as OP, farther on the ground plane, and parallel to the ground line, after having drawn *hi*, repreſenting HI; I have no occaſion to meaſure the diſtance of this line from the ground line, but having taken the diſtance KM, of theſe parallel lines from one another, I ſet it off on the ground line from *q* to *r*; and drawing G*r*, ſo as to cut DC in *k*, I draw *op* through *k* parallel to *hi*, and that is the line required.

If I chuſe to draw a number of theſe parallel lines and at the ſame diſtance from one another, as in Fig. 5, I continue to ſet the ſame diſtance from C to *a*, from *a* to *c*, from *c* to *e*, &c. along the ground line AB, and drawing the lines G*a*, G*c*, G*e*, &c. I find their interſections with the perpendicular CD at *b*, *d*, *f*, *h*, &c. then drawing lines through thoſe points, parallel to the ground line AB, I have their perſpective ſituations. [D]

If I chuſe the diſtances of theſe lines to be unequal, I ſet off unequal diviſions along the ground line.

In this manner may the diſtances be fixed, at which any lines upon the ground plane, parallel to the ground line, may be drawn; and when a method hath been explained of cutting theſe parallel lines, in any length required, then all ſuch lines as *cd*, Fig. 2, may be determined.

SECTION II.

To draw the perſpective ſituation of right lines perpendicular *to the ground line.*

IF I have occaſion to find the perſpective place of a line that is *perpendicular* to the ground line, as *di*, Fig. 2, I meaſure its diſtance from the perpendicular CE, and ſetting it from C, upon the ground line, I join that point, and the point of ſight D, which gives the ſituation required.

For example; in Fig. 4, if I would draw the perſpective appearance of a line, QD, which is perpendicular to the ground line AB, and parallel to CD; I take the diſtance QC, and ſet it off C to *q*; and then D*q*. is the line required. For the ſame reaſon, D*r* repreſents the line DR; alſo D*s* repreſents DS; and D*t* repreſents DT, all drawn at equal diſtances, and parallel to one another.

Since all lines perpendicular to the ground line, when infinitely produced, ſeem to meet on the perſpective plane in the point of ſight D, it is called the *vaniſhing point* of thoſe lines. [E]

SECTION III.

To draw the perſpective ſituation of lines oblique *to the ground line.*

WHen the line, whoſe perſpective repreſentation I want to fix, is *oblique* to the ground line, as *ml*, Fig. 2, and CI, Fig. 6. I meaſure the degree of obliquity by the angle it makes with a perpendicular (as CE) to the ground line; and whatever that degree is, I make the angle at E with the perpendicular CE equal to it, and on the ſame ſide; and to the point in which the line containing the angle falls upon the horizontal line, I draw the line required; for that is the vaniſhing point of the given line, and of all its parallels.

Thus ſuppoſing the angle DCI, in the plane X, Fig. 6, to be 30 degrees, I make the angle DE*i*, in the plane Y, on the ſame ſide of the perpendicular, of the ſame quantity,

tity, and *i* will be the vaniſhing point of the line CI; *i. e.* it will be the point in which that line, infinitely produced, will ſeem to meet the horizon, and alſo that in which all its parallels will ſeem to meet it.

N. B. If it be more convenient (as ſuppoſing E to be without the bounds of the paper) the tangent of the angle of declination from the perpendicular may be ſet off along the horizontal line, from the point of ſight D, the diſtance DE being made the radius.

In Fig. 7, a number of parallels are drawn to the ſame point *a*, 30 degrees to the right hand of the perpendicular CD; and the points in which they meet the ground line are equidiſtant from one another. [F]

PART IV.

To draw the perſpective ſituation of lines not *ſituated* upon the ground plane.

ALL lines not ſituated upon the ground plane are either perpendicular, parallel, or oblique to it; and thoſe that are parallel may be drawn by the help of thoſe that are perpendicular to it; ſo that there remain only two varieties in the lines that are the ſubject of this chapter.

SECTION I.

To draw lines perpendicular *to the ground plane.*

IF the line I have to draw be perpendicular to the ground plane, as *ce*, *df*, *ib*, and *lk*, Fig. 2. and HK in the plane *x* raiſed upright, Fig. 6. it muſt be repreſented by a line perpendicular to the ground line, wherever it is ſituated. If, therefore, the ſeat of the line on the perſpective plane be given, as at *h*, in the plane Y, Fig. 6, repreſenting H in the ground plane X, I raiſe the perpendicular *hk*, and ſomewhere in that line continued will the line required terminate. [G]

SECTION II.

To draw the perſpective ſituation of lines oblique to the ground plane.

IF a line, whoſe perſpective ſituation I want, has an elevation above the ground plane, but no declination from the perpendicular; making DG equal to DF radius, I ſet the tangent of it from D to *a*, Fig. 9, in the perpendicular CE; or, which produces the ſame thing, I make the angle DG*a* equal to the elevation above the horizon (in this inſtance 15 degrees) and *a* will be the vaniſhing point of the line required, and alſo of all other lines parallel to it, as *ga*, *fa*, *ea*, *ba*, *ca*, *da*; all which repreſent lines ſtanding upon the ground line, parallel to one another, having the ſame elevation above the horizon, and no declination from the perpendicular.

If the line, whoſe perſpective ſituation is wanted, be both oblique to the ground plane, and have a declination from the perpendicular, as *mk*, Fig. 2, or CK, Fig. 6; I firſt meaſure the degree of declination (*e.g.* 30 degrees) and making the angle DE*i* equal to it, find *i* in the horizontal line, which is that point in which the line upon the ground plane, perpendicularly under the given line, would meet the horizon, and, through *i*, draw the line *ig* perpendicular to FG. Then taking *iy* equal to *i*E, I make the angle *iyg* equal to the elevation (in this caſe 20 degrees) which gives *g* for the vaniſhing point of the given line CK; and, conſequently (joining C and *g*) C*g* will be the line required. [H]

In Fig. 10, the line C*a* is made to decline 25 degrees from the perpendicular CE, to the right hand; and to have a depreſſion of 15 degrees below the ground plane of the picture; and parallels are drawn to it at the diſtance of one diviſion from each other.

The depreſſion is made in the ſame manner as the elevation, the angle being taken on the contrary ſide of the horizontal line.

In Fig. 11, the line C*a* declines from the perpendicular 20 degrees, it has an elevation of 10 degrees, and the parallels are drawn at equal diſtances.

PART V.

To fix points in perſpective lines, or the doctrine of perſpective meaſures.

IN the two preceding parts, I have given directions for drawing perſpective lines, and their parallels, in all poſſible directions. All that remains to be done, in the whole buſineſs of this art, is to *divide* theſe lines, ſo as to intercept any lengths that may be required in them. For ſince, as was obſerved before, all objects are contained in, or bounded by lines; if we can find the ſituation or direction of thoſe lines, and cut them in any length or proportion that is required, we can draw the outlines of any given objects.

By the interſection of lines, *points* may be fixed in every poſſible ſituation; and *curves* are drawn by finding a ſufficient number of points, and joining them with a ſteady hand. If, indeed, it ſhould happen,

pen, that the perſpective curve is a circle, it will be moſt conveniently drawn by the help of a pair of compaſſes.

SECTION I.

To divide any line lying upon the ground plane, in any proportion required.

CASE I.

To divide a perſpective line lying parallel *to the ground line.*

DIVIDE the ground line, and lay a ruler from thoſe diviſions to the point of diſtance in the horizon. Thus, in Fig. 4, let *hi* be the perſpective line given; if from any point in it, as *u*, I would cut off any particular length, *e. g.* two diviſions, I firſt draw the line G*u*, and continuing it to C, ſet off two meaſures from C to *s*; and then drawing G*s*, I cut the line *ui* at *n* in the proportion required; for *un* will be two meaſures, correſponding to C*s*.

In the ſame manner is *ba* in Fig. 12, made equal *de*, or three diviſions, and *bc* to two diviſions.

Lines parallel to the ground line may alſo be divided by marking the diviſions upon the ground line, as before, and drawing lines to them from the point of ſight D. Thus, if in Fig. 4, I draw D*s* through *u*, and ſet off two diviſions from *ſ* to *t*, the line D*t* will go through the point *n*, making *un* equal to two diviſions, as before.

N. B. In this caſe, and in all that follow, if from any point, as *u*, I want a line of any given length, and the line from which it is to be cut be not previouſly drawn; I muſt, firſt of all, draw, with a black lead pencil, or ſomething equivalent to it, a *faint ſtroke* in the direction required; and when I have found the exact length, muſt take out the reſt. [I]

CASE II.

To divide a perſpective line that is perpendicular *to the ground line.*

LET DC, Fig. 4, be the perpendicular propoſed, and let it be required to cut off *m*C equal to two diviſions. To do this, I ſet off C*q* equal to two diviſions, and draw the line G*q*, cutting the line DC in *m*; making *m*C equal to two diviſions, as was required.

If, from *l*, in the line D*t*, which is another perpendicular to the ground line, I want to cut off two diviſions towards *t*, I draw the line G*l*, and continue it till it cuts the ground line, which is here at C; and ſetting two diviſions from C to *s*, draw G*s*, which cuts the line D*t* in *n*, the point required. [K]

All the perpendiculars D*d*, D*c*, DC, D*a* and D*b*, in Fig. 13, are divided in the very ſame manner; the diviſions being ſet off from the place where thoſe lines meet the ground line, and other lines being drawn from them to the point of diſtance F or G; and this figure will ſhow, that it is a matter of indifference which of the points F or G be made uſe of for this purpoſe. It is only more convenient to take that point of diſtance which is on the ſame ſide with the line to be divided. Thus, if I would divide the line D*b*, I ſet the diviſions from *b* towards *a*, C and *d*; and interſect it by drawing G*a*, GC, G*c* and G*d*. But if I had wanted to divide the line D*d*, I ſhould have ſet off the diviſions the contrary way, and have made the interſections from F.

CASE III.

To divide a line that is oblique *to the ground line.*

LET C*i*, in Fig. 6, be the line given, and let it be ſuppoſed to make an angle of thirty degrees with the perpendicular CD. If I want to cut off ſeven diviſions from the point C, I ſet ſeven diviſions from C to *b*; then, taking the diſtance from the vaniſhing point *i* of this line, to the point of diſtance E in the perpendicular DE, and ſetting it off along the horizontal line to *y*, I draw the line *yb*, which will interſect the line C*i* in *b*, the point required; C*b* being equal to ſeven diviſions.

If from *b* I have occaſion to cut off any number of diviſions more, I ſet them off from *b* towards A; and, drawing lines from *y* to thoſe diviſions, I find the points required.

In the ſame manner may any other parallels to this line be interſected; and if they are to be interſected in a ſimilar manner, the ſame line will ſerve for them all, as in Fig. 14, where the line *ad*, *a*C, and *ab*, are all divided by lines drawn from the point *x* to the diviſions marked upon the ground line C, *b*, and *e*. [L]

If I want nothing but a ſingle point in a given ſituation upon the ground plane, I muſt find its place in the perſpective plane, by ſuppoſing it to be the interſection of two lines, the ſituation of which is given;

as by ſuppoſing one of them to be drawn through it perpendicular to the ground line, and the other to be a line drawn from the point of diſtance, and interſecting it at a given diſtance from the ground line. Thus if I want to find the perſpective place of the point U, in the plane X, Fig. 4 (as if I have occaſion to fix the foot of a column in that place) I meaſure the diſtance UM from the perpendicular CD; and ſetting it off from C to *s*, I draw D*s*, and conclude, that this line muſt go through the perſpective place of the given point U; becauſe every point in this line D*s* is at the diſtance of the point U from the perpendicular CE. I then meaſure US, the diſtance of the point U from the ground line, and ſetting it off from C to *s* I draw GC, which will cut the line D*s* in the point *u* required.

Otherwiſe, this point may be determined by ſuppoſing one of theſe interſecting lines to be parallel to the ground line; for then I meaſure the diſtance US from the ground line, and, by a preceding rule, draw the line *bi* parallel to the ground line. Then meaſuring the diſtance UM from the perpendicular, I ſet it off from C to *s*, and drawing the line D*s* to interſect the line *hi*, I find the point *u* required.

It is evident that any point upon the ground plane might, alſo, be found, by drawing lines making *any given angle* with the perpendicular, and interſecting them at proper diſtances from the ground line; but the methods here deſcribed are much eaſier.

SECTION II.

To divide lines not lying upon the ground plane.

CASE I.

To divide a line perpendicular to the ground plane.

IF *ac*, Fig. 15, be the image of a line perpendicular to the ground plane, and I have occaſion to cut off from the point *a* in it any number of diviſions; for example 5, I take at random any point *d* in the horizontal line, and generally (as being more convenient) beyond the bounds of the picture; and from thence draw a line *de* to any point *e*, taken at pleaſure in the ground line. I then ſet off 5 diviſions from *e* to *f*, in the perpendicular *ek*, and draw the line *fd*. After this I draw a line from the foot *a* of my object, parallel to the ground line, cutting *de* in the point *i*; and raiſing a perpendicular from *i* to the line *df*, find *b*; and this line *ib* contains the 5 meaſures required; which I, therefore, ſet off from *a* to *b* in the given line *ac*. Thus *ab* is equal to *fe*.

The line *de* and *ek*, being once drawn, will ſerve to meaſure any other lines perpendicular to the ground plane, always drawing a line parallel to the ground line from the foot of the object to be meaſured, to the line *de*, and then proceeding in the method deſcribed above.

In order to meaſure objects with the greater exactneſs, the point *d* ſhould be taken at a conſiderable diſtance from the perpendicular *ek*; but the meaſures will be the very ſame at whatever diſtance it be taken.

If a conſiderable number of lines are to be raiſed, all of the ſame height, ſtanding in a right line, a line drawn from the vaniſhing point of that line, to the top of any of them, will touch the tops of them all, which fixes their height with very little trouble. For their tops, being all in a line parallel to the line that goes through their baſes, a line going through them muſt have the ſame vaniſhing point as if it had been ſituated upon the ground plane.

Thus the lines *a*, *b*, *c*, *d*, *e*, *f*, &c. Fig. 16, repreſent rods of equal heights ſtanding upon a line, the vaniſhing point of which is at *x*.

Alſo, if a number of objects be of the ſame height in different parts of the picture; yet, if they all ſtand upon a line parallel to the ground line, they muſt all be drawn of the ſame length. All thoſe marked *d*, in this figure, are examples of it. [M]

CASE II.

To meaſure a line oblique *to the ground plane.*

LET the line be C*g*, Fig. 6, and let it be ſuppoſed that there be occaſion to cut off 8 diviſions from the point C.

Draw CN parallel to the line *yg*, and conſidering CN as a new ground line, lay the given number of diviſions, viz. 8, from C to *a*; join the points *a* and *y* with a line, cutting the given one in the point *k*; and C*k* is the part required. Or,

From the vaniſhing point *g* of the given line C*g* take *gy*, parallel to the ground line AB, and equal to *gy*; on the ground line itſelf ſet off the given length from C to [a] and a line joining [a] and [y] will cut off the required part C*k* as before. [N]

The laſt method will be found moſt convenient in general, not only on account of its being more correſpondent with the rules given in the preceding ſections; but alſo, that in this caſe, we do not encumber that part of the picture, where the images are drawn, with unneceſſary lines and points.

In this example, the given line C*g* ſtands upon the ground line AB; but the rules above will ſerve for lines ſtanding on any part of the ground plane. Should not the application immediately appear to the learner, he may take the following example:

Suppoſe C*g*, Fig. 8, to be an indefinite line, ſtanding on the ground plane at C, and let it be propoſed to cut off a part of it, from C, equal to a given length, for inſtance, 10 diviſions. Find the point [y] by the directions given above; and from C, parallel to the ground line, draw C*b*; then conſidering C*b* as the image of a line parallel to the ground line, cut off from C, by Part 5. Caſe 1. a part C*a* equal the given length, viz. 10 diviſions, and a line joining [y] and *a* will cut the given line C*g* in *k*, the point ſought.

If there be occaſion to make ſeveral other diviſions in the given line C*g*; in order to ſave the trouble of reducing thoſe diviſions to the line C*b*, make C*b* equal the abſolute length of 10 diviſions, and thro' *k*, draw *bk*, cutting *gy* produced, in *d*; and *d* will be a meaſuring point for the line C*g*; ſo that the diviſions themſelves may be ſet upon C*b* at once, and lines drawn from thence to *d*, will cut the given line C*b* in the deſired points.

N B. The point *d* may be found at once, by taking *gd* to *gy*, as the height of the eye is to the perpendicular CL.

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PART VI.

A ſummary account of all the eſſential rules for drawing in perſpective.

AS I have laid down the rules for drawing in perſpective at full length, and have illuſtrated them by a conſiderable number of examples, it may be uſeful to the learner to have a *general view* of what is eſſential to this art, or a ſummary of all the rules, reduced into a ſmall compaſs. This, therefore, I ſhall endeavour to do for him.

All lines that are parallel to the ground line, or perpendicular to the ground plane, in ſhort, all lines that are parallel to the perſpective plane, muſt be drawn parallel to each other. All other parallel lines meet, or have *vaniſhing points* in ſome part of that plane. If the lines lie in any direction upon the ground plane, they will vaniſh ſomewhere in the horizontal line; which is, therefore, called *the vaniſhing line* of that whole plane. If the lines be perpendicular to the ground line, they vaniſh in the point of ſight; but if they be oblique to it, or have a declination from the perpendicular, the angle of this obliquity, or declination, muſt be ſet off from the point of diſtance on the perpendicular, and it will find the vaniſhing point on the horizontal line.

If the line to be drawn be not in the ground plane, but have an elevation above, or a depreſſion below it, ſet off the angle of elevation, or depreſſion, from the point of diſtance, on the horizontal line, and it will find the vaniſhing point on the perpendicular. If the line have both an elevation or depreſſion, and likewiſe a declination from the perpendicular, ſet off the angle of declination as before, and through the vaniſhing point of declination ſo found, draw a line at right angles to the horizontal line; lay the extent between this vaniſhing point and the point of diſtance on the perpendicular, along the horizontal line; and from the point laſt found, ſet off the given angle of elevation or depreſſion, which will cut the line, croſſing the horizontal line, at right angles, in the vaniſhing point required.

All the meaſures of lines upon the ground plane are to be laid down upon the ground line, and the *meaſuring point* of all lines parallel to the ground line, is either of the points of diſtance on the horizontal line, or the point of ſight. The meaſuring point of any line, perpendicular to the ground line, is in the point of diſtance, on the horizontal line; and the meaſuring point of a line oblique to the ground line, is found by extending the compaſſes from the vaniſhing point of that line to the point of diſtance on the perpendicular, and ſetting it off on the horizontal line.

A line perpendicular to the ground plane is meaſured by a triangle, as was particularly explained, p. 31, Fig. 15. Laſtly, a line oblique to the ground plane is meaſured by drawing new horizontal and ground lines from the vaniſhing point and foot of the given line reſpectively, and taking a meaſuring point in this new horizontal line; ſuch, that its diſtance from the vaniſhing point, be to the diſtance of the eye from the ſame point, as the height of the eye is to a perpendicular let fall from the foot of the given line, to the original horizontal line; then lines drawn from this meaſuring point, will transfer diviſions, ſet on the new ground line, to the given image, or the contrary.

* * *

PART VII.

Some more particular directions, to facilitate the practice of drawing in particular caſes.

SECTION I.

Of drawing circles.

I Have obſerved that all curve lines may be drawn by finding a number of points in their circumference, and joining them with a ſteady hand; and this might be deemed ſufficient in a treatiſe, which propoſes to contain nothing but the neceſſary rules, familiarly explained; but, ſince circles are lines that frequently occur in drawing, I ſhall give ſome more particular directions concerning them.

A circle is drawn with the moſt eaſe by firſt drawing *a ſquare* in which it is inſcribed. If the projection of the ſquare be an oblong *trapezium*, the circle will have an oblong appearance, that is, it will be an *oval,*

oval, or more properly, as geometricians have demonſtrated, an *ellipſis*. Having therefore found the image of a ſquare circumſcribing the given circle, draw therein two diagonals, and from their interſection draw lines to the two vaniſhing points of the ſides of the ſquare, cutting the ſides of the projected ſquare in four points, thro' which the image of the circle will paſs. Theſe will be ſufficient, in general, to enable a perſon to draw that image with tolerable accuracy.

Let *abcd*, Fig. 17, be the image of the ſquare, in which it is required to inſcribe a circle. Having drawn the diagonals *ad* and *bc*, and found their interſection at *e*, I draw lines through *e* to *f* and *g*, the vaniſhing points of the ſides (one of them making an angle of 40, and the other an angle of 50 degrees with the ground line) and through the points in which theſe lines interſect the ſides I draw the curve.

If two of the ſides of the ſquare be parallel to the ground line, as in Fig. 18, and conſequently have no vaniſhing points, I interſect the other ſides, that vaniſh in the point of ſight C, by a line parallel to the ground line, as *ef*.

If the figure be large, and very great accuracy be requiſite, the images of a greater number of points in the circumference muſt be found; and by increaſing the number of thoſe points, the curve may be determined to the greateſt nicety.

If I want to draw the appearance of a number of circles, ſuſpended perpendicularly one above another, or different ſections of an upright cylinder, I firſt draw the image *a* of a ſquare with a circle inſcribed, Fig. 19, lying upon the ground; then, raiſing the perpendiculars *e*, *f*, *g*, and *h*, I make another ſquare *b*, at whatever height is required. This ſquare, being raiſed ſo much above the ground plane, will appear very narrow, and conſequently the circle inſcribed in it will be very elliptical. If I had made another ſquare and circle, at the height of the eye, in D, they would both have appeared as a right line.

Advancing above the horizon, I draw the ſquare *c*, the lines which contain it being ſtill drawn to the point of ſight D; but as this ſquare and circle are above the horizontal line, it is the under-ſide of them that we ſee; and as the ſquare and circle *d* are ſtill higher than *c*, they are dilated ſtill more; and the higher I carry theſe ſquares and circles, the leſs oval, and the more nearly circular will their appearance be.

Another example of ſquares and circles, in a different poſition from thoſe laſt deſcribed, is exhibited, Fig. 20. The edges of theſe reſt upon the ground plane, and the whole figure is oblique to the ground line, making an angle of 15 degrees with it. As the drawing of a figure like this may be thought difficult by beginners, I ſhall briefly deſcribe the whole proceſs.

Having pitched upon a point on the ground plane *d*, where I would chuſe the neareſt angle in the figure to ſtand; I draw the line *de* to a point in the horizon out of this picture, making at E an angle of 75 degrees (the complement of 15) with the perpendicular DE. In order to cut this line at right angles by another line upon the ground plane, I find the point *f*, by making the angle DE*f* equal 15 degrees, the complement of 75 to 90. In order to cut as much of the line *fd* as I want for the ſide of the ſquare (ſuppoſe 4 meaſures) I take *fb* equal to *f*E and thereby find *b* the meaſuring point of the line *fd*, and of all its parallels, ſeveral of which will be wanted in this figure. Setting off 4 diviſions upon the ground line, from *x* (the place where a line drawn through *b* and *d* would touch the ground line) I make *dg* equal to 4 meaſures. Then raiſing perpendiculars from *d*, and *g*, I make the line *dh* equal 4 diviſions; and, drawing the line *fh*, the firſt ſquare is compleated. This done, I draw lines from all the angles of it *ij*, *hk*, *gl*, to the ſame vaniſhing point with *de*; for theſe lines will determine the ſize of all the ſquares; and now I have nothing to do but to ſet off on the ground line, from *y*, the place where a line drawn through *a* and *d* would touch it, the diſtances at which I would place the ſquares from one another (ſuppoſe 4 diviſions.)

Theſe I mark upon the line *de*, from the meaſuring point *a*; and when I have drawn the lines *mq*, *nr*, &c. to the vaniſhing point *f*, and have raiſed perpendiculars from the points *m*, *q*, *n*, *r*, &c. to the lines *bk*, and *ij*, the ſquares are completed; after which the circles will be inſcribed in them with eaſe.

All this may be done in leſs time than the deſcription of it can be tranſcribed; and with as little trouble may the ſame number of circles, in any other direction whatever, be drawn.

Cylinders, being figures terminated at both extremities by circles, they may be drawn by firſt making a parallelopiped of the ſame length, and inſcribing circles in the ſquares of each end of it; after which, the line which belongs to the parallelopiped, being drawn with a black lead pencil only, may be wiped out.

If the image of a diameter of any circle, ſituated any where on the ground plane, be given, the image of that circle may be determined in the following methods.

CASE I.

When the given image is parallel to the ground line.

LET *ab*, Fig. 21, be the given image; biſect it in *s*, and through *s*, and the point of ſight C, and points of diſtance D, G, draw 1*s*C, *ts*G, 2*s*D; from either point of diſtance, D, draw lines, D*b*, D*a*, cutting 1*s*C, in 1 and 4, and 14 is a diameter perpendicular to the ground line. Lay down the diſtances GE, DE on the horizontal line to F and H, by which, through *a* and *b*, draw F*al*, F*b*3 and H*b*2, H*a*5 cutting *ts*G and 2*s*F, in *t*, 3, and 2,5 reſpectively; connect the points 1, 2, *b*, 3, 4, 5, *a*, *t*, with a curve, which will be the image required. [O]

CASE II.

When the given line is oblique *to the ground line.*

FIND the image of a diameter which is parallel to the ground line, and then proceed as in Caſe I.

Thus, let [ab] be the given image; continue it to the horizontal line at [g]; take the diſtance [gE,] and lay it on the horizontal line to [d] the meaſuring point of [ab]; from [d] thro' [a] and [b] draw lines cutting the ground line in [e, t]; biſect [et] in [l] and join [ld] cutting [ab] in [h] the image of the center of the circle; then through [h] draw [os] parallel to the ground line, terminating in [de] [dt] and [os] is the image of a diameter of the ſame circle, parallel to the ground line.

By theſe rules, the image of a circle, lying in any known plane, may be deſcribed from the image of a diameter being given. An example of one, in a plane perpendicular to the ground plane, is given in the figure; where *bgm* is the vaniſhing line of the plane and *cd* the given diameter, thus making *gb*, *gr* = gC, and *bm*, *rk* = *b*C, *r*C; then through *e*, the middle of *cd*, draw *gef*, *rneq* and *bpet*; draw *bc*, *bd* cutting *gf* in *o* and *f*; draw *kc*, *kd* cutting *qnr* in *n* and *q*; laſtly draw *mc*, *md* cutting *tpb* in *p* and *t*: and the points *c*, *t*, *f*, *q*, *d*, *p*, *o*, *n*, connected, will give the required image.

The methods delivered above become very uſeful when the images of a great number of parallel circles are to be drawn; for the vaniſhing points C, D, G, F, H, ſerve for drawing the repreſentation of circles in all planes parallel to the ground plane; and *g*, *k*, *r*, *b*, *m*, for all thoſe lying in planes parallel to the plane *cfdo*.

SECTION II.

Of different ground planes.

HITHERTO all the objects to be drawn were ſuppoſed to ſtand upon the ſame ground plane, whoſe vaniſhing line is the horizon; but in drawing landſcapes, and other things, it ſometimes happens, that there are other uniform planes, of conſiderable extent, on which objects are to be drawn. In this caſe, it will be convenient to draw the vaniſhing lines of thoſe planes, and make uſe of them as horizontal lines, for the objects that are to be deſcribed upon them.

Thus, in Fig. 22, the plane *a*, *a*, *a*, *a*, is the common ground plane, the vaniſhing line of which is the horizontal line, paſſing through D; and to this point all lines upon the ground plane, perpendicular to the ground line, are to be drawn. But the plane *b*, has a depreſſion of 15 degrees below the horizon. Its vaniſhing line, therefore, paſſes through H, DFH, being

15 degrees, and the plane *ccc* has an elevation of 15 degrees above the horizon. Its vaniſhing line, therefore, paſſes through I, the angle DFI, being 15 degrees.

If the plane have a declination with reſpect to the perpendicular, or be obliquely ſituated, with reſpect to the ground plane, its vaniſhing line may be found by finding the vaniſhing points of two lines, drawn on that plane in any direction, provided they are not parallel to one another; for the vaniſhing line required, will paſs through both thoſe vaniſhing points.

* * *

PART VIII.

A method of drawing in perſpective from the ichnography of objects.

IF any perſon will take the pains to draw the *ichnography*, or ground plan of what he propoſes to draw, expreſſing the true proportions of all the objects he introduces into it, it will be eaſy, afterwards, to reduce it into true perſpective. This method I ſhall illuſtrate by an example; which will render it very eaſy.

Let the object to be drawn be the triangle *abc*, Fig. 23; and let its ſituation, with reſpect to the grouud line, be the ſame that it has in this figure, being only placed on the oppoſite ſide of it, and therefore in an *inverted poſition*.

To find the vaniſhing point of any of the ſides of this triangle, as [ac] I produce that line, till it meet the ground line in *d*, and draw E*f*, parallel to a*d* by which means I find *f* the vaniſhing point required;

ſo that, drawing *fd*, I conclude that the perſpective repreſentation of the line [ac] muſt be a part of it. In order to determine in what part of this line either of the extremities, as [c] is to be fixed, I lay a ruler from [c] to E, which cuts the line *fd* in *c* the point required. In like manner, the perſpective place of [a] will be found to be in *a*.

By the ſame proceſs, G will be found to be the vaniſhing point of the ſide [bc] and *b* the perſpective place of [b.] Theſe three points being joined, the triangle is completed. But there is no occaſion to find any thing more than the directions of the lines *fd*, and *e*G, together with the points *b* and *c*, for the interſections of the lines will find the point *a*.

The vaniſhing point of the line [bc] might have been found by the ſame method, but it was unneceſſary; becauſe it was determined, by joining the points *bc*, which were found as the extremities of the other lines.

If the body to be drawn be a ſolid, I firſt lay down the ground plan, and when I have reduced that into perſpective, I raiſe either perpendiculars, or lines oblique to the ground plane, from any point in the ichnography, according to the rules laid down before.

If I want no more than to fix the perſpective place of a ſingle *point* upon the ground plane, as for inſtance, that of [c] I draw a line through it in whatever direction I pleaſe, to the ground line, as to *d*. Then finding, as before, *f* the vaniſhing point of that line, I lay a ruler from [c] to the point of diſtance E, which cuts that line in *c*, the point required.

But a more ready way of determining the place of a ſingle point as [a] Fig. 24, is to let fall a perpendicular a*b*, to draw the line from *b* to D, the point of ſight (or the vaniſhing point of the line a*b*, which is perpendicular to the ground line) and then, ſetting off, on the ground line *bd* equal to *b*a, to draw a line from *d* to F, the point of diſtance, cutting the other line in *a*, which is the perſpective point required.

The line *ba* may be transferred to the ground line to *c*, as well as to *d*; and from thence a line may be drawn to G, the other point of diſtance on the horizontal line. This figure ſufficiently ſhows, that both the lines *d*F and *c*G will croſs the line *b*D in the very ſame place *a*. [P]

When objects are very complex, conſiſting of a great number of ſides and angles, it may be the moſt convenient to divide the ichnography of them into ſmall ſquares; and having thrown the ſquares into perſpective, to note the termination of every line, by marking its place in the correſponding ſquare. An example, in Fig. 25 (taken from Mr. Emerſon) will make this method perfectly intelligible.

In the lower part of this figure, the ichnography of a piece of fortification is divided by ſquares; and in the upper part of it, thoſe ſquares are thrown into perſpective, and exhibit a view of the ſame piece of fortification; each point being placed in its correſponding ſquare.

This is a very common method of drawing in perſpective, and is called the method of *Reticulation*.

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PART IX.

An eaſy method of drawing in perſpective without previous menſuration, by means of an inſtrument to take the angles under which objects appear.

THE rules before laid down for drawing in perſpective, ſuppoſe a previous knowledge of the real diſtances, magnitudes, and relative poſitions of all the objects that are introduced into the picture. But, in many caſes, a perſon may be ſo ſituated, that this knowledge cannot be obtained; and in many caſes, likewiſe, the labour which this method requires would be exceedingly troubleſome and diſcouraging. I ſhall, therefore, in this part, deſcribe a method of drawing any objects in true perſpective, without moving from the place in which they are viewed. It is alſo a method of drawing that is very ſimple; it requires the knowledge of very few technical terms; and the rationale of it is extremely obvious.

To ſupply the place of actual menſuration, I provide myſelf with an *azimuth quadrant*, a *Siſſon's theodolite*, or any inſtrument by which I can find the elevation of an object, and likewiſe its angle of declination from the perpendicular going through the point of ſight.

Having this inſtrument, and placing myſelf at what diſtance I think moſt convenient, from any objects that I propoſe to draw; I lay down, upon the paper of my drawing-board, only two lines, croſſing one another at right angles; one of them FG, Fig. 26, to repreſent the horizon, and the other CE the perpendicular, paſſing through the point of ſight D. I alſo chuſe whatever diſtance I think proper to work at, and ſet it off from D to E.

Having thus prepared every thing for the operation, I pitch upon any point in an object before me, as that which correſponds to *x*, and, by the help of my inſtrument, firſt of all, find its declination from the perpendicular to the right or left hand. Suppoſing it to be 10 degrees to the right, I ſet off the angle DE*a* equal to 10 degrees, and conclude, that the point I want to fix muſt be ſomewhere in the perpendicular *ae*. To find in what part of this line the point is, I, in the next place, take its elevation above the horizon, and, ſuppoſing it to be 20 degrees, I make *ab* equal to *a*E, and the angle *abx* equal to 20 degrees, which finds *x* the point required.

In this method may the ſituation of any other point be found, and theſe points, being joined by lines, the whole object will be delineated.

But if the objects to be repreſented contain many *right lines*, that are parallel to one another, ſuch as occur in buildings, machines, &c. there will be no occaſion to take many points; becauſe the ſituations of the lines may be determined by *vaniſhing points*, found by the help of a few of them. Thus having found *y* in the ſame manner as I found *x*, and knowing that the line *xy* is parallel to the horizon; I produce it till it touches the horizontal line in *c*; which is, therefore, the vaniſhing point for that line, and all that are parallel to it, ſeveral of which are repreſented in this drawing. The diſtance at which theſe parallels are drawn, may either be gueſſed by the eye, or be determined with more accuracy by finding a point at one of their extremities, in the manner juſt now deſcribed.

Alſo, if I know the angle that any line, as *yz*, makes with another line, as *xy*, the ſituation of which is known, I have no occaſion to take any point, in order to determine the direction of it. In this Fig. *xyz* repreſents a right angle, which is that which moſt frequently occurs in buildings, machines, &c. To determine, therefore, the ſituation of the line *yz*, I conſider that *c*, the vaniſhing point of *yx*, makes the angle DE*c* equal to 20 degrees, the complement of which, from 90, is 70. I, therefore, make the angle DEH equal to 70 degrees, and a line EH produced till it meets the horizontal line (in a place without the bounds of this picture) gives the vaniſhing point of *yz*, and of every other line at right angles with the line *xy*. To this point, therefore, I draw the line *yz*, and all the others in that figure that are parallel to it.

The point *x* is to the right hand of the perpendicular, and it has an elevation above the horizon; but it can require no particular inſtruction, or example, to be able, from this, to fix any point to the left hand of the perpendicular, and one that has a depreſſion below the horizon. [Q]

If I would introduce *meaſures* into a drawing made in this manner, or find *a ſcale* for the picture; I meaſure ſome one line in the original, as *kf*. In order to this, from *c*, the vaniſhing point of the line *kf*, I take *cd* equal to *c*E, which gives *d* the meaſuring point of the line; and from this point I draw two lines, one from each extremity of the line *kf* to *ib* in the line that bounds the picture, or any other line drawn parallel to the horizontal line. Then I divide

I divide the line *ib* in the ſame proportion as *kf*; and by this means I get a ſcale, by which I can meaſure any other line in the picture, or inſert in it other objects of any given magnitudes, according to the rules laid down above.

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PART X.

Of orthographical perſpective.

IN the methods of perſpective, the rules of which have been laid down in the preceding parts of this treatiſe, the eye of the ſpectator was ſuppoſed to be placed at a definite diſtance from the perſpective plane; they may, therefore, be called *ſtereographical methods*, in alluſion to the ſtereographic projection of the ſphere, in which the eye has a ſimilar ſituation; or rather, ſince the ſituation of the eye, in the common method of perſpective, is variable, the term *ſcenographic* may be more proper. But there is another method of perſpective, in which the eye is ſuppoſed to be placed at an infinite diſtance from the perſpective plane. It may, therefore, be called *orthographic*, in alluſion to the method of projecting the ſphere orthographically.

This method of perſpective is peculiarly adapted to the deſcription of machines, and other things, the magnitudes of which bear but a ſmall proportion to the diſtance which they are generally viewed.

A clearer idea may, perhaps, be conceived of this method of perſpective, by conſidering what kind of a ſhadow of an object would be projected upon a plane, by the rays of the ſun falling perpendicularly upon it. For the diſtance of the ſun is ſo great, that all its rays may be conſidered as coming parallel to one another.

Since there can be no more than two poſitions of a right line with reſpect to the rays of the ſun, viz. *perpendicular*, or *oblique*, the perſpective of lines in this orthographic method is comprehended in two caſes, and that of angles in another. The poſition of a line, parallel to the rays, does not deſerve to be called a *caſe*, becauſe the image of it is nothing more than a point.

If the line be *perpendicular* to the ſun's rays, *i. e.* to the viſual ray, the image of it upon the perſpective plane will be a line, of exactly the ſame length. Thus if AB, Fig. 27. be a ſection of the perſpective plane, and I would lay down the image of any line, as *ef*, I make *cd* of exactly the ſame length; for it is plain that it would be the ſhadow of the line *ef*; the rays coming in the direction of *ed* and *fe*.

If the line be *oblique* to the viſual ray, as *ce*, I lay it down with the ſame degree of obliquity to the ſection of the perſpective plane. Suppoſing it to be 25 degrees, I make the angle *dce* equal to 25 degrees, and the perpendicular let fall from *e*, the remote extremity of this line will intercept *cd*, the image required: For it is evident that *cd* would be the ſhadow of the line *ce* in that ſituation. In other words, the image of any line is always the co-ſine of the angle of its elevation above the perſpective plane, the line itſelf being conſidered as radius.

I ſhall illuſtrate this method of drawing in perſpective by examples, in which both the methods may be compared.

Fig. 28 repreſents a front view of a cube, drawn in the common method of perſpective. It ſtands upon the ground plane, and its upper ſide is terminated by lines drawn to a vaniſhing point in the horizontal line. Only two of the ſides are viſible in this poſition. The image of a circle, inſcribed on one of its ſides is a circle, that of the other an ellipſe.

In the orthographic method, the viſual rays being equally perpendicular to every part of the ſide that is oppoſed to it, all that can appear is a plain ſquare, with a circle inſcribed, as in Fig. 29.

Fig. 30 repreſents the ſame cube in the common method of perſpective, in a ſituation oblique to the eye, one of the ſides being placed at an angle of 25 degrees, and conſequently the other at an angle of 65 degrees. In this poſition three of the ſides appear, and the images of all the circles are ellipſes.

Fig. 31 repreſents the cube, in the ſame ſituation, viewed orthographically. The ſide *cd* was found by drawing *ce*, Fig. 27, equal to the true length of the ſide, making the angle *dce* equal 25 degrees (being the angle that this ſide was ſuppoſed to make with the viſual rays) and letting fall the perpendicular *ed*; ſo as to intercept the line *cd*, the projected length required. The length of the line *bc*, Fig. 31, was found by taking the line *de*, Fig. 27, the ſine of the angle of elevation of the former line. It might have been found by placing the line *ce* at an angle of 65 degrees, and taking the co-ſine of the elevation, as before. For, reverſing the figure, the line *ce* will be ſeen to be in that poſition, and *cf* equal to *de* will be the line required.

Biſecting the ſides of the two parallelograms made upon theſe lines, Fig. 31, points are found for the termination of the axes of the ellipſes, into which the inſcribed circles are projected.

Only two of the ſides of the cube are viſible in this poſition, becauſe one line in the figure, viz. *ch*, is perpendicular to the viſual ray.

If the poſition be changed, and *dx* be conſidered as the baſe, it will repreſent the cube reſting on that ſingle line; the line *cb* being drawn forwards; ſo that *cd*, or *bx* will make an angle of 25 degrees with the perpendicular; the conſequence of which will be, that the upper ſide of the cube *cbyh* will come into view.

If the figure be ſuppoſed to reſt on the line *yb*, it will repreſent the cube leaning ſtill more forwards, and more of the upper ſide *hcxd* in view.

Inſtead of the cube being ſuppoſed to reſt on a ſingle line, in theſe caſes, we may conceive it to be viewed obliquely, the eye being raiſed above the plane on which it ſtands; ſo that the upper ſide of the cube may be ſeen, without its being raiſed towards the eye.

If I would repreſent three ſides of this cube as ſeen orthographically, it is evident that it muſt both ſtand obliquely, as in Fig. 31; and alſo that the eye muſt be elevated above the plane on which it ſtands. Suppoſe then, every thing elſe in this figure remaining the ſame, the eye be raiſed 10 degrees, as in Fig. 32, it will be evident, that the lines *cb* and *dx* muſt be made ſhorter than the correſponding lines in the preceding figure, for being inclined 10 degrees to the viſual ray, they muſt be leſſened in the proportion of the co-ſine of that angle to radius, and reduced from the length *ci*, Fig. 27, to *cj*. Alſo every line in the figure, parallel to them, muſt be made of the ſame length.

The lines *cd* and *cb* muſt now no longer be one line, for the upper ſide of the cube will be ſeen, and conſequently the baſe would be ſeen too, were the figure tranſparent.

In order to determine the reduced lengths of the ſides *cd* and *cb*, and at the ſame time the angle they make with each other, in this ſituation, I lay down the ſides and angles DC*b*, Fig. 33, in their real ſize and poſition, and draw C*e*, from the point C, in the ſituation in which a plane would paſs through the eye and that point, in the poſition given before, *i. e.* ſo as to make the angle *e*Cb equal 25 degrees, and *e*CD equal 65. Perpendicular to C*e*, and through the point *b* I draw the line *be*, and produce it till it meet the line CD, continued, in *f*. Upon the center *e*, with the radius eC, I deſcribe the arc C*b*, and from *b* I ſet off *bg* equal to the angle of the elevation of the eye. Then parallel to *eb*, I draw *gc*, cutting the perpendicular C*e* in *c*, the point into which C will be projected. Joining *cb*, I have the image of the ſide C*b*, and drawing D*d* parallel to C*e*, to cut *fc* in *d*, I get *dc* for the other ſide. Alſo the angle *dcb*, is the projected angle required. [R]

Accordingly, in Fig. 32, I make *dcb* the ſame angle as that in Fig. 33, contained under the ſame ſides; and completing all the parallelograms, the projection of the whole figure is determined.

If the line *cd* be not drawn parallel to the ground line, the angle it makes with it, added to the angle *ncb*, muſt always make 10 degrees, or any other angle that is required.

Inſtead of ſuppoſing the eye to be raiſed above the plane of the baſe, the image would be the ſame, if the cube were ſuppoſed to reſt upon the corner *c*, the line *cb* leaning forwards, in an angle of 10 degrees.

It may be convenient to apply this method of orthographic perſpective in drawing very *complex figures*, when it would require a good deal of time to find the vaniſhing points of all the ſides. If a view in common perſpective be neceſſary, it will be eaſy to reduce this orthographic projection to it by the rules laid down, in Part 8, to draw the perſpective appearance of objects from their *ichnography*, which is given by this orthographic method.

In drawing machines in general, there is no occaſion to lay down angles and ſides with this exactneſs. It will be ſufficient to lay down the ichnography of the object to be deſcribed; as for example, the parallelogram, in Fig. 34, in whatever poſition may be thought moſt convenient, and drawing the elevation in perpendiculars over the baſe. Thus in Fig. 35, a hollow parallelopiped, or trough, is repreſented ſtanding upon the baſe of Fig. 34. A figure like this is perfectly intelligible, and much more eaſy for a workman to copy after than any other drawing whatever; becauſe all the dimenſions are taken from a common ſcale, and the imagination may be ſufficiently aſſiſted to conceive of it by good ſhading. It might not, perhaps, be amiſs for perſons who are learning to draw in perſpective, to begin with this ſimple method, and accuſtom themſelves to it for ſome time, before they attempt any other. [S]

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PART XI.

Of ſhadows.

WHEN objects are ſeen in a ſtrong light iſſuing from one place, they project *ſhadows*, which it is very convenient, at leaſt ornamental, to draw in the perſpective views of them; I propoſe, therefore, in this part, to lay down rules for delineating theſe ſhadows, beginning with thoſe which are projected on the ground plane on which the objects ſtand, as being the moſt common, and the moſt uſeful caſe.

SECTION I.

To draw the ſhadows of bodies on the ſame planes on which they are ſituated.

SINCE rays of light always iſſue in ſtraight lines, it is evident that the ſhadow of any object will cover juſt as much ſpace, as it would hide from an eye ſituated in the place of the luminous body. For the ſame reaſon, the ſhadow of any point of an object muſt be ſomewhere in a right line drawn on the ground plane, from a point perpendicularly under the luminous body, through the point that is perpendicularly under that whoſe ſhadow is required; and the preciſe point of this line on which the ſhadow will fall, will be determined by a right line drawn from the luminous body through the point of the object.

This obſervation ſupplies a rule for projecting the ſhadows of all bodies whatever. Find the point upon the ground plane that is perpendicularly under the luminous body. From this draw right lines through perpendiculars let fall from thoſe points, the ſhadows of which are required, and interſect them by right lines drawn from the luminous body through thoſe points.

Thus, in Fig. 36, if the luminous body be at *a*, perpendicularly over *b*, upon the ground plane, and it be required to find where the ſhadow of *c*, the top of the line *cd* (in one angle of the ſolid [x]) will fall;

ſince *cd* is a line perpendicular to the horizon, I draw a right line through *b* and *d*; and drawing another right line through *a* and *c*, interſect it in *e*; which will, therefore, be the place of the ſhadow required. Finding, in the ſame manner, *f* the place of the ſhadow of *g*, and *h* the place of the ſhadow of *i*, and joining the points *d*, *e*, *f* and *h*, I have the out-lines of the ſhadow of the whole ſolid [x].

If I want the ſhadow of the rod [y] which ſtands oblique to the ground plane, I let fall a perpendicular *kl* from the top of it, *k*, to the ground plane, and drawing the lines *ak* and *bl*, I find that they interſect each other at *m*; which, therefore, marks the place of the ſhadow of *k*; and ſince the rod is a ſtraight one, and conſequently all the points of the ſhadow muſt fall on the ſame right line, I draw a line from *d*, the foot of the rod, to *m*, and thus get the ſhadow of the whole rod.

In the ſame manner may be drawn the ſhadows of ſolid bodies, the baſes of which project beyond the perpendiculars let fall from them. Firſt find the place where the perpendicular would fall upon the ground plane within the ſolid, and proceed with it, as if nothing ſolid had been in the caſe.

If the ſhadows of objects be made by the light of the ſun, the ſituation of that luminary, with reſpect to the picture, muſt be determined, and this is done by the help of the following conſiderations.

By reaſon of the immenſe diſtance of the ſun, it muſt always be ſuppoſed to be in, or over ſome point in the horizontal line, in which the ground plane, when extended to an infinite diſtance, is conceived to vaniſh; and the particular point in the horizontal line muſt be determined from the number of degrees that the ſun is ſituated to the right or left hand of the perpendicular that goes through the point of ſight; in the ſame manner as the vaniſhing point of a right line upon the ground plane, that is oblique to the ground line, is found.

If, for inſtance, the ſun be 30 degrees to the left hand of it, I make the angle DEH, Fig. 37, equal to 30 degrees, and H will be that point in the horizontal line over which the ſun may be ſuppoſed to be perpendicular; and nothing is now wanting to fix its preciſe place above the horizon, but to know its *altitude*, which is laid down in the ſame manner as the vaniſhing point of a line that is elevated above the ground plane. If, for inſtance, the ſun be 35 degrees high, I make HI equal to HE; and, raiſing the perpendicular HK, make the angle HIK equal to 35 degrees, and K will be the ſun's place.

If, now, I would draw the ſhadow of any object made by the ſun in this ſituation, I ſuppoſe K to be the place of the candle *a*, in Fig. 34, and H to be *b*, the point upon the ground plane that is perpendicularly under it, and I proceed exactly as in that example. Thus, to draw the ſhadow of the ſolid [x] Fig. 36, the place of the ſun being at K, I draw H*a*, and interſecting it by K*b* in *c*, I have the place of the ſhadow of *b*; and joining *ac*, I have the ſhadow of the perpendicular line *ab*. In like manner I find *d*, the ſhadow of *f*, at the top of the line *ef*; and alſo *g*, the ſhadow of the vertex of the line *hi*. Then, joining theſe points, I have the ſhadow of the whole object [x]: for it is evident, from the poſition of it, that the ſhadow of any other line in the figure muſt fall within that of theſe.

When, as in this example, the place of the ſun is fixed above the horizontal line, it is evident, that the ſhadows will be projected as falling towards the ſpectator, and that they will always be larger than the objects. To draw the ſhadows which are made by the ſun, having the ſame degree of elevation *behind* the ſpectator, which will make the ſhadows fall nearer the horizontal line, and leſs than the objects; the perpendicular from H, Fig. 37, muſt be let fall to L, and the angle HIL muſt be the ſame as HIK in the former caſe; and then, if the point L be made uſe of, inſtead of K; as theſe points are ſituated on the contrary ſides of the horizontal line, the ſhadows of objects will be repreſented as they would be made by the ſun placed on the back of the ſpectator; and this poſition of ſhadows is generally thought more agreeable in a picture than the other.

To find, in this manner, the ſhadow of the line *ab*, in the ſolid [x] Fig. 39, I draw H*a* to the foot of it, and L*b* to the top, interſecting each other in *c*, the place of the ſhadow. Finding, in the ſame manner, where the ſhadow of the line *ef*, behind the body, would fall, I join that point and *c*, by a line which cuts the ſide of the figure in *d*, where only the ſhadow begins to be viſible. Thus joining *a*, *c*, and *d*, the ſhadow of the ſolid [x] is completed.

In both theſe caſes, the ſun is ſuppoſed to be ſituated either *before*, or *behind* the picture. If it be *in the plane* of the picture, it is evident that its place muſt be in the horizontal line, at an infinite diſtance from the point of ſight. Conſequently all lines proceeding from it may be ſuppoſed parallel to one another. To draw the ſhadows of objects made by the ſun in this ſituation, draw lines parallel to the horizontal line from the foot of the object, and interſect them by lines making angles with the horizontal line equal to the ſun's altitude.

This practice is very eaſy by means of the drawing board and ſquare. Thus if I would draw the ſhadow of a ſolid [x] Fig. 40; with the ruler of the ſquare parallel to the ground line AB, I draw *ac*; and fixing the moveable tranſverſe of the ruler in the angle of the ſun's elevation, I draw the line *bc*, which gives *c* for the place of the ſhadow for *b*. In the ſame manner alſo, I find *d* the place of the ſhadow of *e*; and joining the points *dc*, I get the ſhadow of the whole figure; though it is ſo ſituated, that part of it is hid by the object. If there be ever ſo many lines, the ſhadows of which are to be found, once fixing of the ruler ſerves for them all. I have nothing to do but to ſlide it along the drawing-board, till I come to the points from which the lines are to be drawn, both for the lines that are parallel to the horizon, and thoſe that interſect them.

To draw the ſhadow of the ſolid [y] Fig. 40, which lies oblique, both to the horizon, and to the ground plane, I draw *ac* from the foot of the line *ab*, parallel to the horizon; and fixing the moveable tranſverſe of the ruler, ſo as to make an angle equal, to the ſun's altitude, I draw *bc*. In the ſame manner I alſo find *g*, the ſhadow of *f*; then drawing *gc*, and joining *cd*, the place where the neareſt extremity of the ſolid touches the ground, I am able to compleat the outline of the whole ſhadow; for the ſhadows of all the other lines muſt fall within theſe.

Let it be obſerved, that it makes no difference in the ſhadow, whatever be the height, or ſhape of objects, provided their tops, and every part of them, be in the ſame right lines proceeding from the luminous body. Thus the ſhadows of the ſeveral objects *a*, *b*, *c*, and *d*, Fig. 41, perfectly coincide, from the plane where each of them begins.

SECTION II.

Of ſhadows intercepted by other objects.

WHEN the ſhadow of a line falls upon any object, it muſt neceſſarily take the form of that object. If it fall upon another plane, it will be a right line, and if upon a globe, or cylinder, it will be circular.

If the body intercepting it be a plane, whatever be the ſituation of it, the ſhadow falling upon it might be found by producing that plane till it intercepted the perpendicular let fall upon it from the luminous body; for then a line drawn from that point would determine the ſhadow, juſt as if no other plane had been concerned.

But the appearance of all theſe ſhadows may be drawn with leſs trouble, by firſt drawing it through theſe interpoſed objects, as if they had not been in the way, and then making the ſhadow to aſcend perpendicularly

dicularly up every perpendicular plane, and obliquely on thoſe that are ſituated obliquely, in the manner that I ſhall now deſcribe.

Finding that the ſhadow of the upright pole *a*, Fig. 42, would have reached, in a ſtraight line, to *b*, if it had not been intercepted by the object [x]; I firſt draw the ſhadow through that object, and finding that it firſt touches it at *c*, where it muſt aſcend perpendicularly to the ground line, I raiſe the perpendicular *cd*; and ſince the upper ſurface of this body is a plane, parallel to the ground plane, I make that part of the ſhadow *de* parallel to the reſt of the ſhadow upon the ground plane.

If the firſt ſide of the interpoſing object that the ſhadow falls upon be hid from the eye, as in Fig. 43, I ſtill draw the line *ab*, as if it were viſible, and where the ſhadow meets with it, raiſe the perpendicular *bg*, as before. Or, I may begin at the termination of the ſhadow, and tracing it back to the place *c*, where it cuts the interpoſing body, find the point *f*, where it left that new ſurface, and draw the part *fg* of the ſhadow parallel to the reſt of it, as before.

If the ſurface of the body interpoſed be an inclined plane, I find, by perpendiculars, as *a* and *b*, Fig. 44, both where the ſhadow enters the oblique ſurface, at *e*, and where it leaves it at *d*; and then joining the points *c* and *d*, I have the direction of the ſhadow on that oblique plane.

If theſe planes change ever ſo often, I have only to draw perpendiculars from them all, and the direction of the ſhadow will be found with great eaſe.

If I want a ſhadow of only part of an object, or an object not ſtanding upon the ground, as for inſtance, the part [ab] Fig. 44, I ſuppoſe it to be continued to the ground, and I find where the ſhadow of [b] falls, as at *e*, in the ſame manner as if the pillar had been no higher; and then the part at *f*, lying upon the ground plane, together with the part *de*, upon the inclined plane, will be all the ſhadow of [ab] required.

If the pillar reach no higher than [b] and it had been required to find on what part of the inclined plane its ſhadow would terminate; I ſuppoſe the object to have been continued higher, till ſome part of the ſhadow had fallen upon the ground plane, as at *f*; then drawing the perpendicular *bd*, I find *d*, the place where the ſhadow would have left the inclined plane, if the object had been continued; and the point *e*, where a line from [b] cuts the line *dc*, is the termination of the ſhadow required.

If the object upon which the ſhadow is thrown be circular, the ſhadow will be circular. It muſt, therefore, be drawn with juſt ſuch a degree of curvature, as the body itſelf would have had, if it had been interſected by a plane in the direction of the ſhadow. Thus, in Fig. 45, when the ſhadow of the pole comes to the cylinder, lying obliquely to the horizon, it goes over it, in the form of an ellipſe, ſuch as would have been made, if the cylinder had been interſected by a plane in the direction of the ſhadow. The curvature may be drawn with ſufficient accuracy, by obſerving where the ſhadow cuts the line repreſenting the baſe of the cylinder, and raiſing a perpendicular on that point, equal to the cylinder's apparent diameter, for this will find the place where the ſhadow goes over it. Thus, the ſhadow croſſing the baſe of the cylinder at *a*, I raiſe the perpendicular *ac*, cutting the line repreſenting the higheſt part of the cylinder in *c*, and the ſhadow will paſs over that point.

SECTION III.

Of faint ſhadows.

WHEN objects are not ſuppoſed to be viewed by the light of the ſun, or of a candle, but only in the light of a cloudy day, or in a room into which the ſun does not ſhine, there is no ſenſible ſhadow of the upper part of the object, and the lower part only makes the neighbouring parts of the ground on which it ſtands a little darker than the reſt. This imperfect, obſcure kind of ſhadow is eaſily made, being nothing more than a ſhade on the ground, oppoſite to the ſide on which the light is ſuppoſed to come; and it may be continued to a greater or leſs diſtance, according to the ſuppoſed brightneſs of the light by which it is made. It is in this manner (in order to ſave trouble, and ſometimes to prevent confuſion) that the ſhadows in the greater part of drawings are made. Examples of theſe ſhadows may be ſeen in Fig. 15, 16, 26, and ſome others in this work.

SECTION IV.

To draw the reflected images of objects in water.

THE appearance of the image of any object, as viewed in a plain mirrour, is exactly of the ſame dimenſions with the image itſelf, but inverted with reſpect to it. If, therefore, from the foot of the repreſentation of any object, ſtanding near the water ſide, a ſimilar image be drawn downwards, in an inverted poſition, of the ſame length and form with the repreſentation itſelf, the part that falls upon the water is the reflected image, or as much of it as can be ſeen in that ſituation.

Thus *b*, Fig. 46, is the reflected image of the object *a*, ſtanding cloſe to the water ſide; *c* is as much of the image of *d*, ſtanding upon the plane, at ſome diſtance from the water, as can be ſeen in that poſition; and *e* is as much as can be ſeen of the image of *f*, ſtanding upon an eminence near the water.

If the object be oblique to the ground plane, as *h*, give the image *g* an equal degree of obliquity, and that will exhibit the true image required.

Theſe objects are uniform throughout. When objects of other forms are to be repreſented, let it be remembered, that, ſince theſe images muſt be *inverted*, that part which is higheſt in the object muſt be loweſt in the image, and *vice verſa*. Thus the tops of trees, and the heads of men muſt be drawn the fartheſt from the brink of the water.

PART XII.

General advices and directions, relating to the art of drawing in perſpective.

IT diſcourages many perſons, who would be glad to learn the art of perſpective, to think that they muſt be obliged to meaſure every thing that they draw; and, it is true, that all the rules of perſpective, except thoſe laid down in Part ix. do ſuppoſe the objects to have been meaſured, and no drawing can be perfectly accurate without it. But where extreme accuracy is not required (as indeed it very ſeldom is) there is little occaſion to meaſure any thing; and yet a perſon who has a juſt idea of the nature of perſpective, will make a drawing infinitely more juſt, and agreeable, than another perſon can, who ſhall even meaſure every thing.

This is more eſpecially the caſe where a great number of parallel lines, cutting each other at right angles, or any given angles, are to be repreſented; ſuch as always occur in drawing buildings, machines, furniture of houſes, &c. For ſince all parallel lines, not parallel to the picture, have the ſame vaniſhing points, if a perſon only know where to fix thoſe points, ſo that lines drawn from them ſhall make any given angle with one another, his drawing, by the help of thoſe points, will be far more agreeable to truth, and look infinitely better, than if he had gone to work without that previous knowledge to guide him. Beſides a perſpective drawing is always made in far leſs time than any other.

A perſon who underſtands this art, has nothing to do but to judge by his eye of the proportion of objects, and all his lines are ſure to be perfectly true upon that ſuppoſition; and, therefore, the whole will be conſiſtent with itſelf. But the random deſigner may, in one draught, make a hundred different ſuppoſitions; and as it is a great chance if any two of them agree, the whole will be very inconſiſtent with itſelf, and, therefore, muſt make a very aukward appearance.

This conſideration, I think, is a matter of conſequence for the encouragement of young deſigners. I ſhall, therefore, give them an example of it, in my own practice.

The drawing, plate vii, which is a perſpective view of an electrical machine of my conſtruction, was made when I had very little knowledge of the theory or practice of perſpective. I only knew how to fix vaniſhing points in the horizontal line, according to any poſition of lines upon the ground plane; but as all the planes in this machine were either perpendicular to the ground plane, or parallel to it, I was ſenſible that I wanted no other; and though I knew but little of the doctrine of perſpective meaſures, I could judge nearly enough of the proportion of the ſeveral lines by my eye, and therefore could do without them.

All that I did, therefore, was to conſider the center of the globe as the point of ſight, and to draw the horizontal line and perpendicular croſſing one another at right angles in it. On the perpendicular

I ſet off the diſtance at which I choſe to work, and from the ſame point ſet off angles of 35 degrees on one ſide, and 55 on the other, which gave me two vaniſhing points, from which I could draw lines cutting one another at right angles. Then obſerving the angle under which the whole machine was viewed in the ſituation I choſe for myſelf, and having the machine before me, I made every thing as nearly in proportion to it as I could judge by my eye. As I meaſured nothing, I drew no ground line, but, when I had done, cloſed the drawing where I thought proper. It is a fault in this drawing, that I fixed the point of diſtance too near the point of ſight. But this doth not make the drawing leſs juſt, or leſs uſeful.

The electrical battery, plate iii, was drawn at the ſame time. Both the ſides have equal degrees of obliquity to the horizontal line; and therefore I drew all the lines to one or other of the points of diſtance (or the tangents of 45 degrees) on each ſide of the point of ſight. Here, alſo, I had no meaſures, or gound line.

With more knowledge of the art of perſpective I might have made more elegant drawings of theſe figures, but theſe are perfectly juſt, and ſufficiently plain. In theſe, and in many other caſes, I have often been agreeably ſurprized to find how much uſeful practical knowledge may be derived from a very little theory.

I could engage to communicate to any perſon all the knowledge that is requiſite to make theſe two drawings in two or three minutes, whereby he might finiſh them, at his leiſure, in little more than an hour each; and without that little knowledge, a perſon might puzzle himſelf a month about them, and, after all, produce nothing that ſhould not be quite prepoſterous.

It is a maxim with deſigners, founded upon experience, that no picture ſhould take in quite ſo much as is comprehended within an angle of 90 degrees. In other words, the figure ſhould not extend to both the points of diſtance on the horizontal line. The reaſon of this rule is, that the eye cannot, at one time, diſtinctly take in ſo great a compaſs, but the extremities of the proſpect will be confuſed and indiſtinct.

In general, the leaſt diſtance ſhould be about one-third more than the height of the eye. In many drawings it ſhould be much greater.

In drawing landſcapes, a low horizon is particularly agreeable. It ſhould be about one-third of the depth of the picture.

The rule laid down above for chuſing the diſtance, ſhould be obſerved even where very high horizons are choſen, in order to give what is commonly called *a bird's view* of objects. Thus [a] Fig. 47, is a view of a plain ſquare incloſure, with a pretty low horizon, and [b] is a view of the ſame ſquare incloſure with a higher horizon, and a proportionably greater diſtance. By this means more of the inſide of the incloſure is ſeen.

I ſhall cloſe this part of the work with the following extracts from Mr. Emerſon's Perſpective.

"If any print or perſpective view, be looked at through a lens, whoſe focal diſtance is equal to the principal ray, and the print placed in its focus; it will be ſo magnified, as to have the very ſame appearance, as at the place it was drawn for.

"Glaſſes which are not of a due focal length, will not give the exact appearance of a place. Shorter glaſſes make the diſtances leſs, and ſo contract the view. And longer glaſſes make the diſtances greater, and extend the view.

"Since glaſſes of a long focal diſtance, give a large and extenſive proſpect of a country, therefore they are better than ſhorter glaſſes. And when the proſpects are well drawn, and properly coloured; it is very delightful to view them through a good glaſs, as they ſo nearly imitate nature. And though there is but one focal length that will give a true appearance, yet the draught will always appear a regular piece of perſpective, though it may not exactly repreſent any place in the world, ſuppoſing the eye placed ſomewhere in the principal ray. And the draught will ſeem longer in proportion to the focal diſtance of the glaſs made uſe of; or in proportion to the apparent diſtance of the neareſt part of the picture.

"As there is nothing more pleaſant than viewing the draughts of countries, towns, cities, magnificent buildings, and other grand objects, when well drawn: to ſee them to the beſt advantage, the focal diſtance of the glaſs ſhould be juſt ſo long, as not to ſhew the ſcratches and coarſeneſs of the engraving; or not much longer; for then the view will be narrow, and the parts too ſmall to be ſeen ſo far off. And if it be far off, it will hide the beauties as well as deformities. And to get a proper glaſs, obſerve at what diſtance the ſcratches diſappear to the naked eye, and that is the focal length of the glaſs. Perſpective views ſhould be drawn, ſo that the point of view be farther off, than is generally practiſed, if you would have the piece to be a true copy of nature. The principal ray ſhould not be leſs than two feet, and then the draught, being looked at through a lens of that focal diſtance, will appear in perfection, and give a true repreſentation of the place it was drawn for. The view ſhould be ſo large as to ſubtend an angle at the lens of about 30 degrees. And it is proper to put the lens in a ſhort ſquare tube; which will confine the ſight, and direct it to the perſpective draught; which, to complete its beauty, ought to be coloured with the ſame colours as the natural objects appear in. For this purpoſe water colours need only be uſed."

PART XIII.

The definition of all the technical terms *made uſe of in this treatiſe.*

THE *ground plane;* the plane on which both the ſpectator, and the objects that are to be drawn, ſtand.

The *perſpective plane;* a plane ſtanding perpendicularly upon the ground plane, interpoſed between the eye and the objects. On this plane (as on a glaſs window) the images of objects are ſuppoſed to be intercepted; ſo that their *perſpective appearance* is the appearance they have on this plane.

The *ground line;* the line on which the perſpective plane is ſuppoſed to reſt.

The *point of ſight;* that point in the perſpective plane which is neareſt to the eye, and at the ſame diſtance from the ground line with the height of the eye above the ground plane. N. B. A line drawn from the eye to the point of ſight is ſometimes called the *principal ray.*

The *horizontal line*; a line upon the perſpective plane, drawn through the point of ſight, parallel to the ground line.

The *perpendicular*; a line on the perſpective plane, drawn through the point of ſight, perpendicular to the ground line and the horizontal line.

Points of diſtance; points on the perſpective plane, ſet off from the point of ſight, ſometimes on the horizontal line, ſometimes on the perpendicular; at the ſame diſtance from the point of ſight that the eye is ſuppoſed to be at from the perſpective plane.

Vaniſhing points; points on the perſpective plane, in which parallel lines, infinitely produced, ſeem to meet.

Meaſuring points; points from which any lines in the perſpective plane are meaſured, by laying a ruler from them to the diviſions laid down upon the ground line.

A
GENERAL VIEW
OF
THE THEORY OF
PERSPECTIVE.

WITH
NOTES,
RELATING TO IT.

A GENERAL VIEW of THE THEORY OF PERSPECTIVE,

To which ſome of the particular DEMONSTRATIONS in the following NOTES refer.

DEFINITION I.

The vaniſhing line *of an original plane, is a right line in the picture, formed by the interſection of an imaginary plane paſſing through the eye, and parallel to the original one, with the picture.*

THUS, in Fig. 4, having raiſed the planes Y and Z, if we ſuppoſe a plane to go through the eye at *a*, and to be parallel to the ground plane X, that plane will cut the picture Y, in the horizontal line FG; which, therefore, is the vaniſhing line of the plane X, or ground plane.

A little attention will make it evident, that no point in the original plane X, can have its image in the picture, above the line FG, ſo long as the poſition of the picture and the eye remain the ſame; and, that if the plane X were infinitely extended beyond the picture, yet its perſpective appearance, in the preſent example, would be a finite ſpace, bounded by the vaniſhing line FG.

DEFINITION II.

The vaniſhing point *of an original line, is that point in the picture, where an imaginary line, drawn from the eye, parallel to the given line, cuts it.*

THE planes Y and Z, Fig. 4, being raiſed as before, the vaniſhing point of the line CD, on the plane X, perpendicular to the picture, will be the point D; for it is plain, that an imaginary line drawn from the eye *a*, to D, is parallel to the line CD, ſince CD = *ba*, and the line *a*D is alſo perpendicular to the picture.

It will not be difficult to conceive, that were the original line CD, extended beyond the picture, to any diſtance whatever, the image of no point in it could appear higher in the picture, than the point D.

PROPOSITION I.

Suppoſing the picture a plane, which may be extended every way at pleaſure, the line *in which any original plane, not parallel to the picture,* cuts *the picture (and which may be called the* interſecting line *of that plane,) is* parallel *to the* vaniſhing line *of the ſame plane.*

FOR, ſince the imaginary plane, paſſing through the eye, and producing the vaniſhing line, is parallel to the original plane, the lines formed by their interſections with any other plane, as the picture, will be parallel (16 Eu. 11.) Thus the ground line AB, Fig. 4, which is the interſecting line of the ground plane, is parallel to the horizontal, or vaniſhing line of that plane.

Corollary. Planes parallel to the picture, can have no vaniſhing line.

For a plane paſſing through the eye parallel to ſuch planes, is alſo parallel to the picture, and therefore can never cut it.

PROPOSITION II.

The image of every original line, perpendicular, or oblique to the picture, tends to its vaniſhing point: and the images of ſuch original lines, as are parallel to the picture, can have no vaniſhing points; they will, therefore, be parallel to their reſpective originals.

LET the planes, Fig 4, be prepared as before; and let any original line, as KM, be produced till it cut the picture in C (which point is called the *interſecting point* of the line KM) join this point and the vaniſhing point D, of the line; then, the appearance of every point in the original line, infinitely extended beyond C, will be found in this line, and conſequently, the image *km* of any part KM thereof, muſt tend to the point D.

A line parallel to the picture, will have the imaginary line drawn from the eye, which ſhould find its vaniſhing point, parallel to the picture alſo; ſo that ſuch a line can have no vaniſhing point. The image of ſuch a line will, therefore, be parallel to this imaginary line drawn from the eye, and, of conſequence, parallel to its original.

Corollary. The images of parallel lines, not parallel to the picture, tend to the ſame point; and parallel lines, parallel alſo to the picture, will have their images parallel one to another.

PROPOSITION III.

The vaniſhing point of every original line, lying in the ſame original plane, will be found in the vaniſhing line of that plane.

FOR the vaniſhing line of a plane, is the boundary of the appearance of that plane, infinitely extended before the eye; and the vaniſhing point of a line, is alſo the boundary of the image of that line, extended in like manner. It is plain, therefore, that the latter muſt be found ſomewhere in the former.

So the vaniſhing point of the line KM, Fig. 4, is a point D, in the vaniſhing line FG, of the plane X, in which it lies.

Corollary I. Hence, if we know the vaniſhing points of two lines, not paralle the one to the other, and which lie in the ſame plane; we know, alſo, the vaniſhing line of that plane.

For a line joining thoſe vaniſhing points, is the vaniſhing line required.

Corollary II. The imaginary line drawn from the eye, which finds the vaniſhing point of any original line, is parallel to the plane in which that line lies.

PROPOSITION IV.

The image of a line, parallel to the picture, will be ſimilarly poſited with its object; *and have its* length *in proportion to* that *of its original, as the* diſtance between the eye and the image, *is to the* diſtance between the eye and the object.

LEt Y, Fig. 48, be the picture, E the place of the eye, AB an original line, parallel to the picture; I ſay, *ab* : AB : : E*b* : EB.

For, *ab* being parallel to AB, (Prop. II.) the triangles E*ab* and EAB are ſimilar; whence, *ab* : AB : : E*b* : EB : : E*a* : EA.

PROPOSITION V.

The image of any point, in a given original line, not parallel to the picture, is a point in the indefinite image of that line, which divides the ſaid indefinite image in the proportion of the diſtance between the eye and vaniſhing point *of the given line, to the* diſtance between the given point and point of interſection.

LEt Y, Fig. 49, be the perſpective plane, E the place of the eye, AB an original line, whoſe vaniſhing and interſecting points are v and A, reſpectively; I ſay, vb : bA : : Ev : Ab.

For, ſince Ev and AB are parallel, (Def. II.) the triangles Evb and BAb are ſimilar; whence vb : bA : : Ev : AB.

NOTES,

Relating, chiefly, to THE THEORY OF PERSPECTIVE.

Note A. Page 8.

IN drawing the few objects compriſed in Fig. 2, there is occaſion to introduce every real variety in the practice of perſpective. It will, therefore, be of uſe to the learner to attend to it; and when he is maſter of all the neceſſary rules, to draw it over frequently, varying the lengths and ſituations of the lines contained in it. For when once a perſon can draw any line in this ſmall picture, independent of the reſt, from the ſituation and meaſure of it, previouſly given, he will be able to draw the repreſentation of any object, or any number of objects whatever, be they ever ſo complex.

B. Page 14.

Since theſe lines are parallel to the ground line, they are parallel to the picture; hence, (per Prop. II.) their images will be parallel to the original, and, therefore, muſt be parallel to the ground line alſo.

This is clearly exemplified in Fig. 4, where OP and HI, upon the ground plane X, are repreſented by *op* and *hi* on the perſpective plane Y. For if the perſpective plane be perforated in *k*, *l*, *m* and *n*, and a ſtraight wire be put through *a* in the plate Z, repreſenting the eye of the obſerver, and alſo through the holes *k* or *l*, it will fall upon OP; and if it be put through the holes *m* and *n*, it will fall upon the lines HI.

C. Page 15.

That this method of determining the diſtances of theſe parallel lines muſt be accurate,

curate, will be evident from conſidering that G, Fig. 4, being ſituated at the height of the eye from the ground line AB, and GD being the diſtance of the eye from the perſpective plane, the line DC may be imagined to be the perſpective plane, ſeen edgeways; and then any point *q*, ſituated three meaſures beyond it, muſt appear at *m*, in a ſtraight line drawn from the eye at G to it: *m*, therefore, is the proper diſtance from C, at which that point in the line required muſt be drawn. Now, ſuppoſing the perſpective plane to revolve, and to be reſtored to its former ſituation, with the eye at *a*; for the ſame reaſon that *m* muſt be placed at the diſtance of *m*C from the ground line AB, every point in the ſame line muſt be placed at the ſame diſtance; and therefore a parallel to the ground line, drawn through *m*, will be the line required.

To explain this more clearly, put wires through *a* and *m*, when the planes Y and Z are ſet upright; and it will be ſeen, that this wire is exactly in the ſame ſituation, and of the ſame length, with the line G*q*;

both of them paſſing through *m*, and cutting off the ſame length, C*m*, from the line CD.

D. Page 16.

The more of theſe lines I draw upon the ground plane, equidiſtant from one another, the nearer they ſeem to approach, when reduced to the perſpective plane; becauſe every interval between them is continually farther from the eye. The lines CA, Fig. 9, being continued *ad infinitum*, and the ſame proceſs repeated, it is plain that every ſucceeding parallel would approach nearer to the preceding one, till at laſt they coincided with the horizontal line FG.

This is evident from Def. II. for the original lines being perpendicular to the ground line, are perpendicular to the picture; therefore, the imaginary line paſſing through the eye, which gives the vaniſhing point of theſe lines, will be perpendicular to the picture alſo; but it muſt be parallel to the ground plane (Cor. II. Prop. III.) conſequently will cut the picture in the point of ſight.

E. Page 18.

That all ſuch lines, perpendicular to the ground line, will meet on the perſpective plane in the point of ſight D, is alſo evident from conſidering the ſituation of the ſpace included between the parallel lines QD and SD, each of which is equally diſtant from the perpendicular CD. The extremity of this ſpace QS, being neareſt to the eye, muſt appear wider than the diſtance between theſe lines at any place more remote from the eye; and the farther off theſe diſtances are taken, the leſs they will appear. Conſequently the ſides which bound this ſpace muſt appear to approach nearer and nearer continually, till they meet ſomewhere; and this point, in which they meet or vaniſh, muſt be ſomewhere in the line CD; becauſe each ſide of the ſpace is every where equidiſtant from the eye.

It muſt, likewiſe, be ſomewhere in the horizontal line FG; for raiſing the planes Y and Z, the point M on the ground plane will appear in *m* on the perſpective plane, to an eye ſituated at *a*; the point K will appear in *k*; and ſo CD on the ground plane, being infinitely continued, and every point in it, more remote from the eye, appearing to riſe higher and higher in the perſpective plane, they muſt at laſt reach as high as D. And they can never riſe higher than D; becauſe, were the viſual ray and the line CD actually parallel, ſtill the former could only paſs through D.

For the ſame reaſon that the lines QD and SD will meet at D on the perſpective plane, the repreſentations of RD, TD, and all other lines that are parallel to them, at whatever diſtance they be taken, on either ſide of the perpendicular CD, muſt meet in the ſame point of ſight D.

F. Page 19.

It is evident, that CI, Fig. 6, being ſituated upon the ground plane, muſt have its vaniſhing point ſomewhere in the horizontal line DF, for the reaſon given before Prop. V. It will alſo be that point in the horizontal line where an imaginary line drawn from the eye, parallel to CI, cuts it (Def. II.) This imaginary line will, therefore, form an angle, at the eye *a*, with the perpendicular *a*D, equal the angle ICD. The part D*i* intercepted between D, and this vaniſhing point muſt, conſequently, be the *tangent* to that angle, the radius being *a*D; or, which is the ſame thing, the radius being DE.

Let the plane Y be interpoſed between the eye at *a* and the line CF upon the ground plane, and it will be evident that the baſe of the infinite triangle (formed by the lines CD and CI) in the true horizon, and the line D*i* upon the perſpective plane, which repreſents it, are equally tangents to the ſame angle, under which they are both viewed; all the difference between them being in the radius of the circle.

Wherever C*i* meets the horizontal line, all parallels to it muſt be drawn to the ſame point, for the reaſon given before (Cor. I. Prop. II.)

To illuſtrate the preceding demonſtration by experiment, let a ſtraight wire paſs from *a* to H, Fig. 6, and it will paſs through the perſpective plane Y in *b*. If a more diſtant point in the line CI be taken, the plane Y will be pierced in the line Ch produced, which ſhows that it tends to vaniſh in *i*.

For the ſame reaſon that the vaniſhing point of Ci, and of all its parallels, is in the horizontal line FG, the vaniſhing points of all lines ſituated upon the ground plane, in whatever direction they be drawn, muſt be ſomewhere in the ſame line. For this reaſon the horizontal line is called the *vaniſhing line* of the ground plane.

And for the ſame reaſon that the ground plane hath its vaniſhing line in the horizon, any other plane, making any angle with the ground plane, will have a vaniſhing line, making the ſame angle with the horizontal line, which the interſecting line of the new plane makes with the ground line; and every operation may be performed with this, as with the firſt ground line, and the firſt horizon. This conſideration throws the greateſt clearneſs into the theory of perſpective, and makes the practice exceeding eaſy; and this great improvement we owe to the ſagacity of Dr. Brook Taylor.

G. Page 21.

Since lines perpendicular to the ground plane are parallel to the picture, their images muſt be parallel one to another and to their originals. (Prop. II.) Thoſe images will, conſequently, be perpendicular to the ground line.

To illuſtrate this, let pins, and other bodies, be fixed upright on the ground plane X, and conſidered as viewed from *a* in Z, while the repreſentation is taken upon the plane Y.

H. Page 22.

Imagine the given line CK to be ſituated in a plane [x] perpendicular to the ground plane, ſuch a plane will cut the ground plane in the line CH, making the angle

DCH equal the given declination of the line CK; the point *i* is, therefore, the vaniſhing point of CH; and, ſince CH lies in the plane [x] as well as in the plane X, the vaniſhing line of the plane [x] will paſs through the point *i* (Prop. III.) Again, the plane [x] being perpendicular to the ground plane, would, if extended, interſect the picture in the line CDE; hence its vaniſhing line will be parallel to DE (Prop. I.) therefore the vaniſhing point of CK will be found in *ig*. We are, laſtly, to prove that *g* is that vaniſhing point.

Since the imaginary line *ag* is parallel to CK (Def. II.) it muſt make the angle *iag* = the angle HCK, or angle of elevation of the given line. The line *ig* is, therefore, the tangent of the angle HCK to radius equal *ai*. But *ig* is the tangent of the angle *iyg* (= HCK) to radius *iy* and *iy* = iE = *ia*. Conſequently, the point *g* is truly determined.

The imagination may be aſſiſted in conceiving of this, by ſuppoſing the horizon to be raiſed as much as the angle of elevation of the given line. For, in this caſe, the repreſentation C*k* of the line produced, muſt as certainly meet this new horizontal line, paſſing through *g*, as any lines upon the ground plane, continued, muſt meet the common horizon. Indeed, upon this ſuppoſition, a plane paſſing through CK, and the foot of the obſerver, would take place of the old ground plane.

I. Page 26.

Raiſe the planes Y and Z, Fig. 4. Then, ſince the given line HI is parallel to the picture, the image *un* of any part UN thereof, will be parallel to its original; whence per Prop. IV. *un* : UN : : *au* : *a*U; *un* : (C*s* =) UN : : G*u* : GC, per conſtruction; but ſince DG = *a*D, G*u* = *au*; and GC = *a*U, therefore the image *un* is juſtly determined, by means of the lines GC, G*s*. Again, we have *un* : (*st* = UN) : : D*u* : D*s*; but D*u* : D*s* : : G*u* : GC; hence, *un* is the image of UN, as found by this method alſo.

By raiſing the moveable plates in Fig. 4, and putting a ſtraight wire through *a*, and the ſeveral perforations in the perſpective plane Y, theſe rules will be occularly demonſtrated. For a ſtraight wire extended from *a* to U, will paſs through *u*, and being extended to M it will paſs through *m*, ſo as to intercept *um* in the perſpective plane, correſponding to UM on the ground plane. And ſince triangles ſtanding upon equal baſes and between the ſame parallels are equal, the triangle *a*UN will be equal to the triangle GC*s*, or D*st*, *a* being the ſame height from the ground plane with D and G. Alſo if theſe equal triangles be cut by a plane parallel to their baſes, as by the plane *hi*, the new baſes will ſtill be equal; ſo that *un*, in this caſe, will be the baſe to both the triangles. For the ſame reaſon, they would have been as well divided by lines drawn from F, the other point of diſtance, as from G.

Univerſally, perſpective meaſures are determined by drawing lines from the points of diſtance, or other meaſuring points, to the meaſures, previouſly laid down upon the ground plane, or to lines ſituated in a ſimilar manner with regard to the perſpective plane.

K. Page 27.

Suppoſing a line to be drawn from *a* to M, it will paſs through the perſpective plane in *m*. But C*q* being equal to CM, and DG equal to *a*D, the triangles *a*CM and G*q*C will be exactly equal; the latter being, in fact, the very ſame with the former, when the lines *a*D and *a*M have been ſuppoſed to revolve, till *a* came into the ſituation of G. Conſequently *m* being an immoveable point during the revolution, its place will be the very ſame in which ever of the triangles it be taken.

In the very ſame manner may it be ſhown, that any other point, as *u* or *n*, will be found to have the ſame place upon the perſpective plane, whether it be fixed by a line drawn through it from the eye to the object, or by a line drawn through it from the point of diſtance in the horizon to a point on the ground line, at the ſame diſtance from the perpendicular with the object itſelf. For by Prop. V. we have D*u* :

$us : : (aD =)\ DG : SU$; but from the ſimilarity of the triangles DuG and suC, it is, as Du is to us, ſo is DG to Cs; but $Cs = SU$ per conſtruction. Therefore the point u is the image of U. The ſame reaſoning will prove, the point n to be the image of N.

L. Page 29.

Since the triangles iyb and bCh are ſimilar, we have $ib : bC : : iy : Ch$. But $Ch = CH$ and $iy = iE = ia$ per conſtruction. Therefore $ib : bC : : ia : CH$; whence per Prop. V. the point b is the image of H, as was required.

M. Page 33.

The proceſs in this ſection hardly needs any demonſtration, it being evident that any row of objects, of the ſame height, ſtanding upon the ground plane, on a line perpendicular or oblique to the ground line, muſt appear to diminiſh till they vaniſh in the horizontal line. Conſequently the triangle *dfe*, Fig. 15, will contain in it perpendicular lines equal to the apparent height of all objects that are of the height of fe; and this will be the caſe wherever the point d is taken in the horizontal line. For $ib : ef :: di : de$; but $di : de :: dm : dl$, that is, as $Ha : HI$; and ſince this proportion will hold wherever the point d be taken in the horizontal line, it follows, that the length of the perpendicular ib will not be altered thereby; it being always to ef in the conſtant proportion of Ha to HI.

N. Page 34.

In the firſt method, we have the triangles kgy and kCa ſimilar; whence $gk : kC :: gy : Ca$; but gy is equal to ga, the diſtance of the eye from the vaniſhing point g, and Ca is equal CK per conſtruction. Therefore $gk : kC :: ga : CK$. Hence (per Prop V.) the point k is the image of K. In the ſecond method, ſince gy is drawn parallel to Ca, we have $gk : kC :: gy : Ca$; but gy and Ca are equal gy and Ca, and of conſequence equal ga and CK reſpectively. Therefore, &c. as before.

O. Page 47.

Let AB, Fig. 21, be the original of *ab*, on which deſcribe the circle AIB IIII, and draw the diameters I, IIII; LIII, IIV, dividing the circle into 8 equal parts. Then it is plain, the diameters I IIII, V II, L III, will have their vaniſhing points at C, D, G; and therefore 1C, 2D, and *l*G are their indefinite images; and ſince A I and IIII B are parallel to V II, they will have the ſame vaniſhing point with V II, viz. D. Therefore, lines drawn from D to *a* and *b* will cut 1C in 1 and 4, the images of I and IIII.

Since F is the meaſuring point of *ls*G, and H the meaſuring point of 2*s*D, lines drawn from F to *a* and *b* will cut off the parts *ls*, *s*3 reſpectively, equal to *as* and *sb*. For the ſame reaſon, lines drawn from H to *a* and *b* will cut off the parts *s*5 *s*2 equal *as*, *sb*. Conſequently, *l*3, and 5 2 are the images of L III and V II, diameters of the original circle A I B IIII. Q. E. D.

P.

P. Page 55.

The demonſtration of this rule is made eaſy by the conſideration of Fig. 6. Let HC, in the plane X, be the line to be determined. The vaniſhing point is fixed by the line *ai*, drawn parallel to HC, or E*i*, which, making the ſame angle with the perpendicular, meets the horizontal line in the ſame place *i*. And the point *h* is determined by drawing a line from H to *a*, the place of the eye. In both theſe caſes the picture is placed between the eye and the object; and if the elevation of the eye, in Fig. 6, occaſion any difficulty to the perſon who compares theſe figures, let him lay the plane Y flat down, ſo as to be a continuation of the plane X, and let him ſuppoſe E and [h] to be viſible through it on the backſide of the paper (as they are marked in the figure for that purpoſe) and he will find that a ruler laid from H to E, will paſs through the point *h*. In this ſituation of the plates X and Y, all the lines have the very ſame poſition to one another as the lines in Fig. 32.

Q. Page 61.

The *rationale* of this practice is very obvious, depending on this one principle, that every line in the perſpective plane makes a tangent to the angle of viſion; and it may be fully illuſtrated by the help of Fig. 6. Let *g*, in the plane Y, be the point in the perſpective plane, repreſenting the point of any object that is required to be drawn. When the planes Z and Y are raiſed, it will be evident that the angle *i*ED is equal to *ia*D. Conſequently *i*, the foot the of perpendicular *ig*, is truly fixed by this rule. Alſo the angle *iyg* is equal to *iag*, ſo that the point *g* is likewiſe truly fixed by it.

R. Page 70.

To demonſtrate this caſe, I ſhall make uſe of a larger figure, Fig. 50.

Let the triangle to be projected be C*bi* in the plane Y, equal to CBI in the plane X. Proceed as in the former example, and both of the triangles will be completed.

pleted by producing the ſides *bi*, and BI to A. If the viſual rays be perpendicular to the perſpective plane Y, and the triangle A*bc* be made to turn upon its baſe AC, as in the plane X, it is evident that, in every poſition of the plane X, the ſhadow or image of the point B will always fall upon ſome part of the line *b*D; and, at whatever angle the eye be elevated above the plane X (ſuppoſe 30 degrees) it is alſo evident, that (raiſing the plane Z) the perpendicular BH will fall upon the ſame point H, in the line *b*D, as the parallel GH (FG being taken = 30 degrees = DBH) for ſince the triangles BDH and GDH have BD equal to GD, DBH equal to DGH, and the angles at H right ones, the whole triangles will be equal; and conſequently the line DH will be the ſame in both; A*b* will, therefore, be projected into AH, C into HC, and the angle A*b*C into AHC.

If the ſide *b*A terminate in *i*, and BA in I; draw *ik* parallel to *b*D, and it will cut HA in K, the place of the image of I.

It is evident from what was advanced before, that the image of the point [I] muſt fall ſomewhere in the line HA: It is equally evident, that it muſt fall ſomewhere in the perpendicular *ik*, in every poſition of the plane X; and therefore can be no where but in the interſection, K, of theſe two lines.

An Addition to PART XII. p. 94.

THAT the young artiſt may be a proper judge of the importance of chuſing a proper *diſtance* in perſpective drawings, I have given a duplicate of Fig. 20. in Fig. 51. In order to take in all the lines and points that were principally requiſite to the deſcription, the diſtance, DE, in the former, was taken very ſmall; in conſequence of which, the more remote ſquares ſeem to be out of proportion, though to an eye in a proper ſituation for viewing them (viz. at the diſtance DE, perpendicularly over the point D) the angles they ſubtend will be found to be equal, and therefore the drawing is juſt. Fig. 51 differs from the former in nothing but the diſtance, which is taken near three times as great. By this means all the ſquares fall within the meaſuring points *a* and *b*, and the diſtortion, obſervable in the former figure, is avoided.

What we call the perſpective plane is only an imaginary thing; for, in reality, we do not refer the objects of ſight to any plane, ſome parts of which are more diſtant from the eye than others. Whenever, therefore, the diſtance in drawing is ſo ſmall, that one part of the picture is conſiderably more remote than another, a diſtortion will be perceived by an eye in the ſituation in which drawings are generally viewed. No drawing can appear perfectly natural except on the ſurface of a ſphere, the eye being placed in the center. But this inconvenience, attending the common perſpective, will be ſufficiently obviated, if the diſtance be taken large, in proportion to the ſize of the figures to be drawn. It is becauſe theſe things are better underſtood by example than precept that I have given the two drawings of theſe figures.

Upon the principle mentioned above, the problem concerning the perſpective diameter of columns, ſtanding at equal diſtances from the ground line, admits of a ſolution; but it is not to my purpose in this treatiſe to diſcuſs ſuch particulars.

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Page 15.	line 5.	*from the top*, *for*, C to *g*, *read*, C to *q*.
30.	18.	*for*, C to *s*, *read*, *s* to C.
34.	14.	*for*, *gy*, *read*, *g* y.
36.	11.	*for*, C *b*, *read*, C *g*.
72.	13.	*dele* [S.]
77.	19.	*for*, 34, *read*, 36.
110.	12—16.	*for* *v*, *read* *a*.
114.	11.	*for*, lines CA, Fig. 9, *read*, line CA, Fig. 5.
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114.		*This is evident*, &c. This paragraph ſhould be at the End of the Note E, and inſerted nex the paragraph, *For the ſame reaſon*, &c. in page 116.

Publiſhed by
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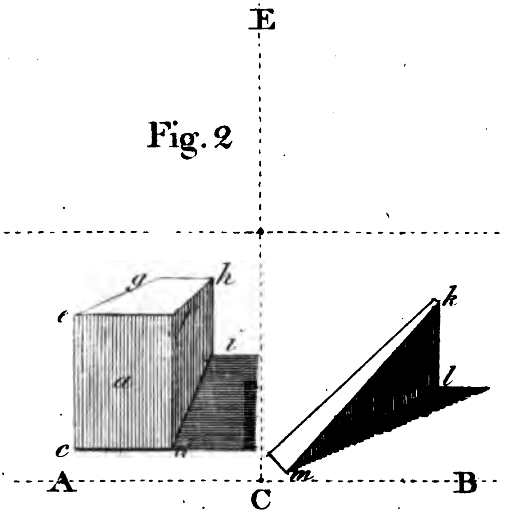

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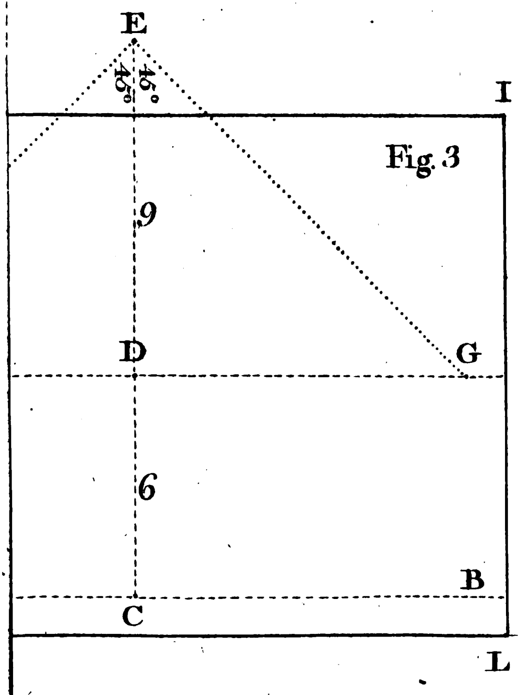

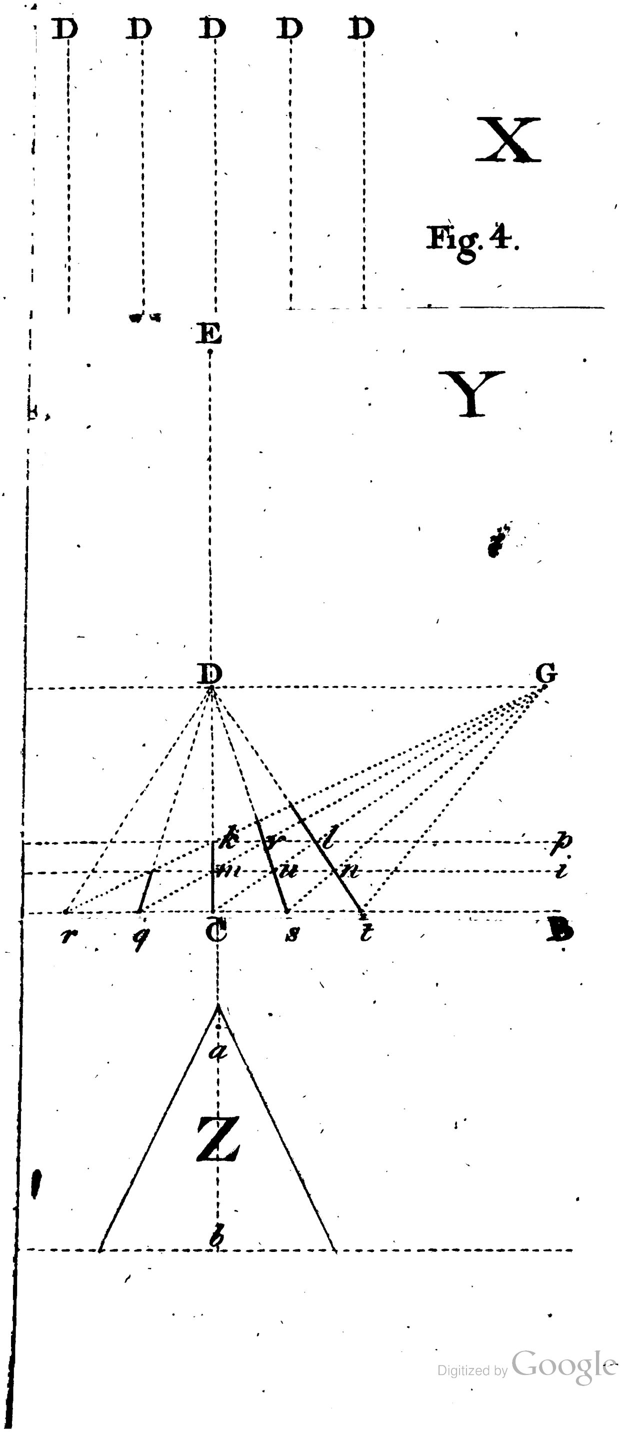

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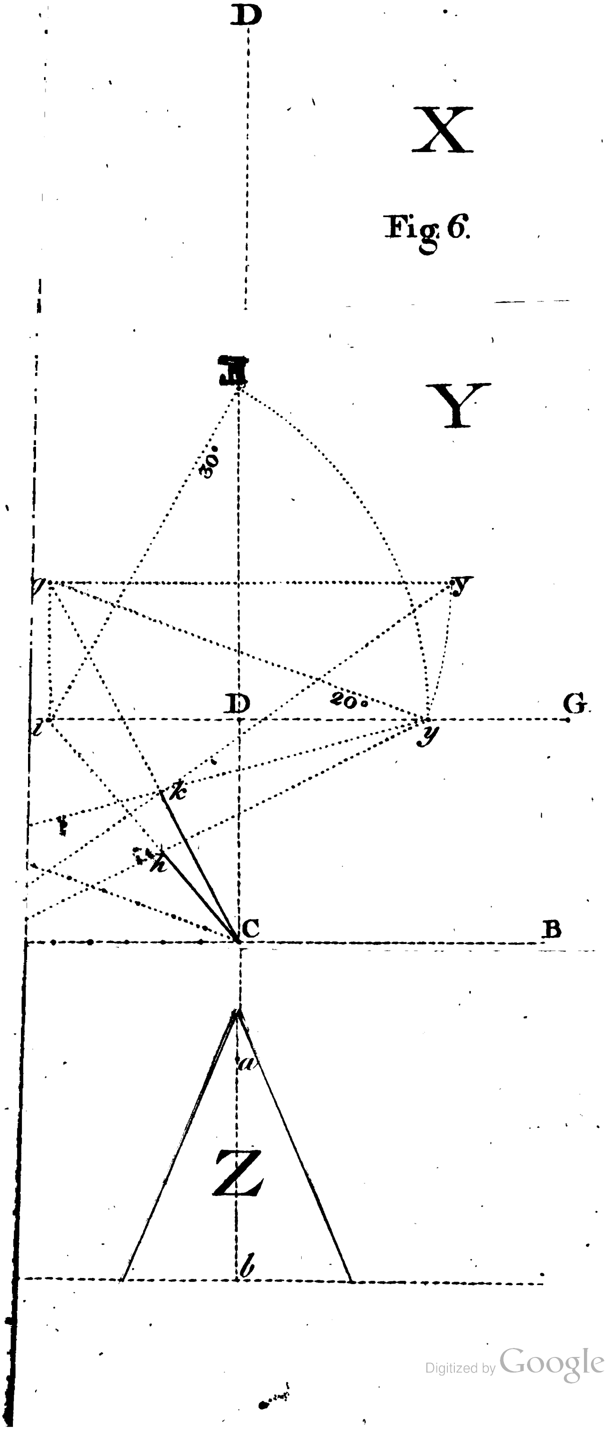

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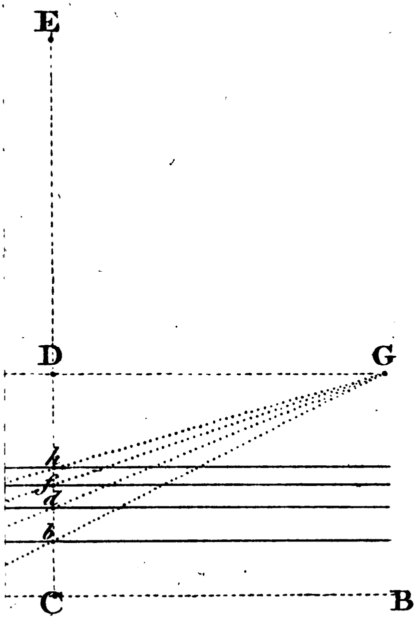

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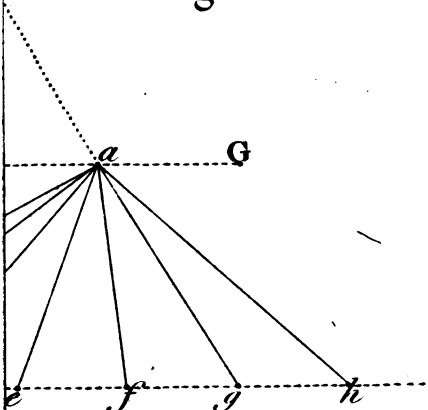

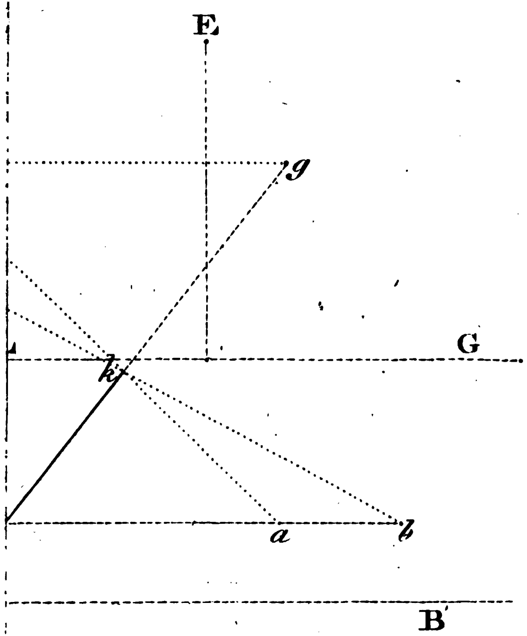
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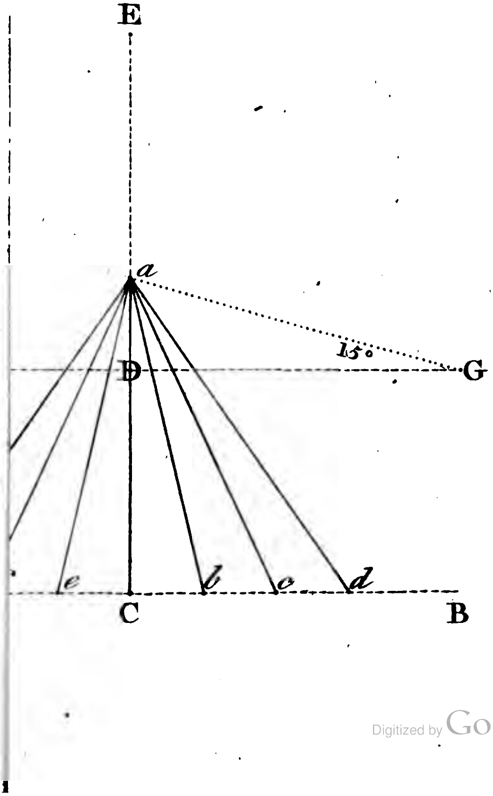
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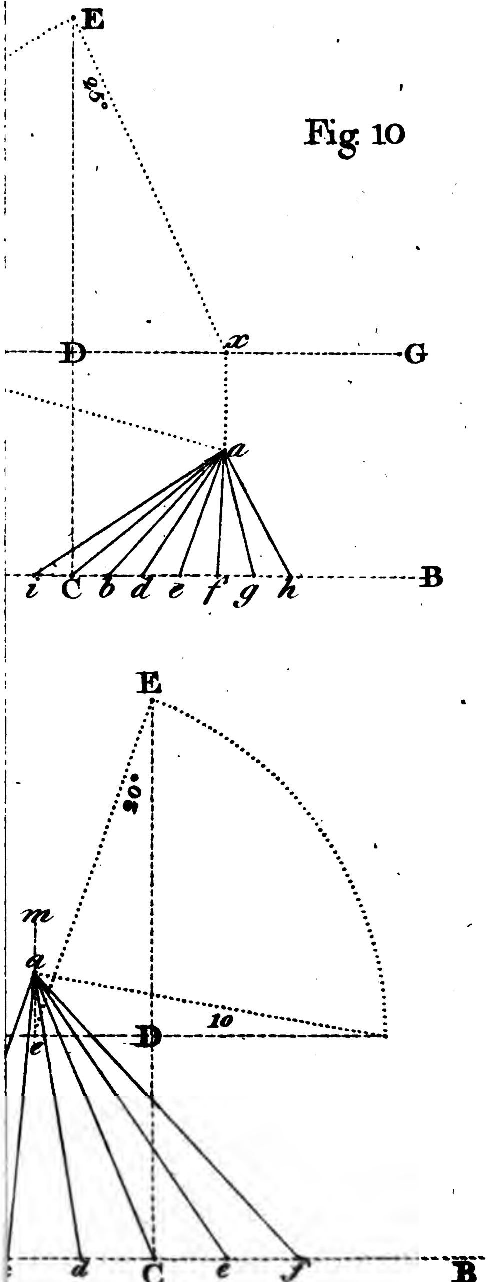

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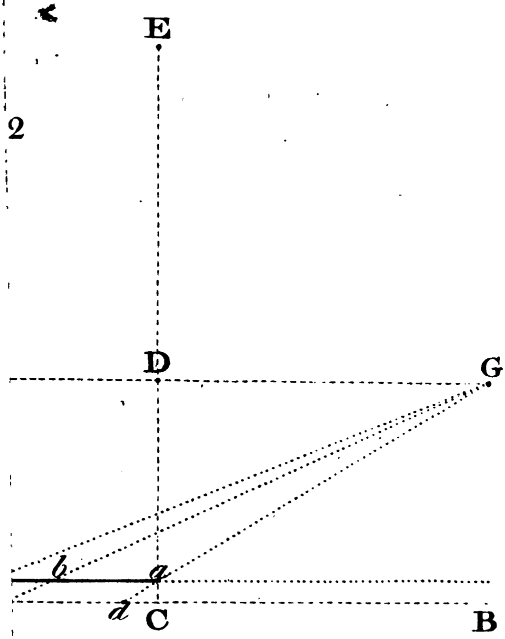

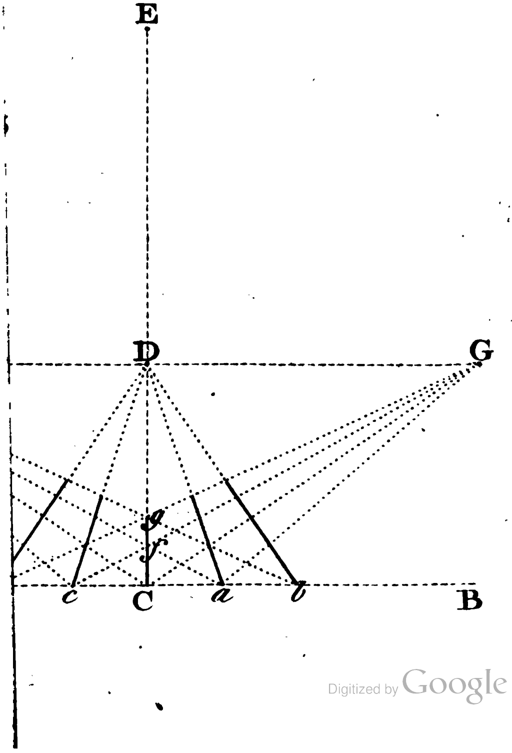

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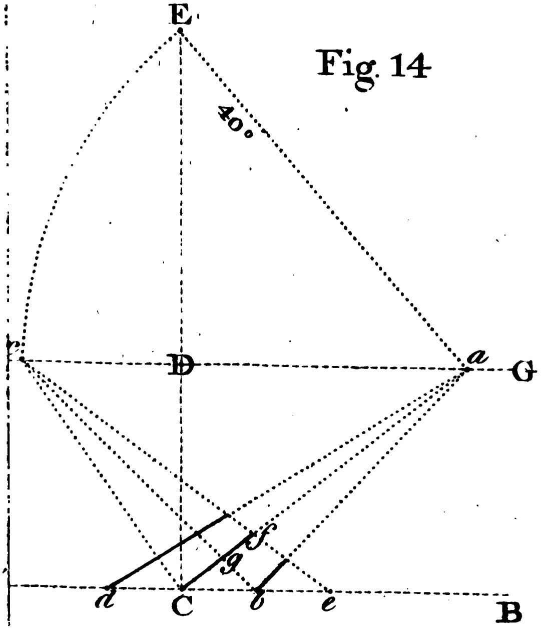

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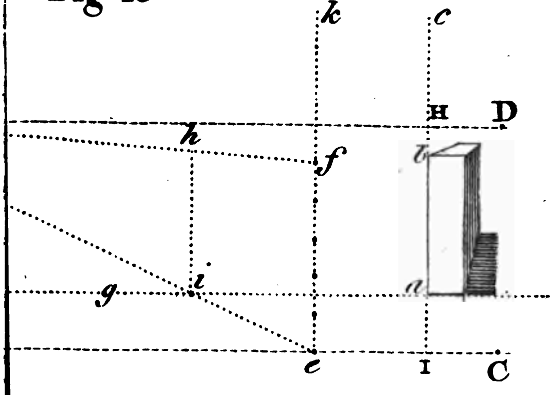

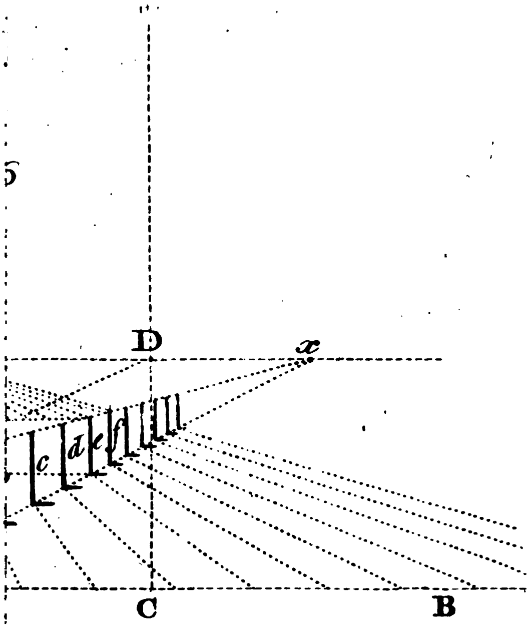

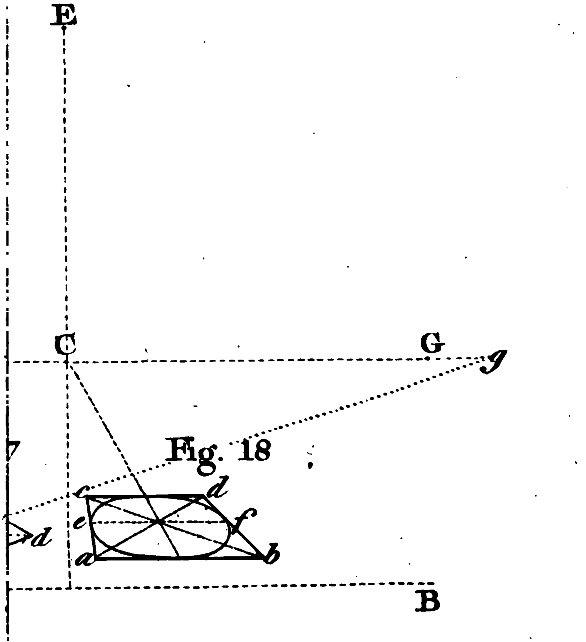
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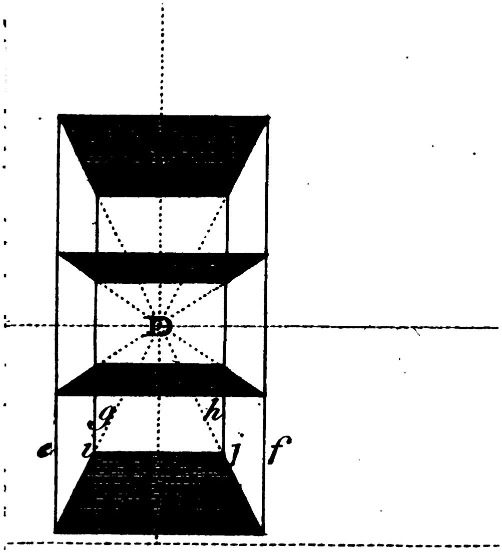

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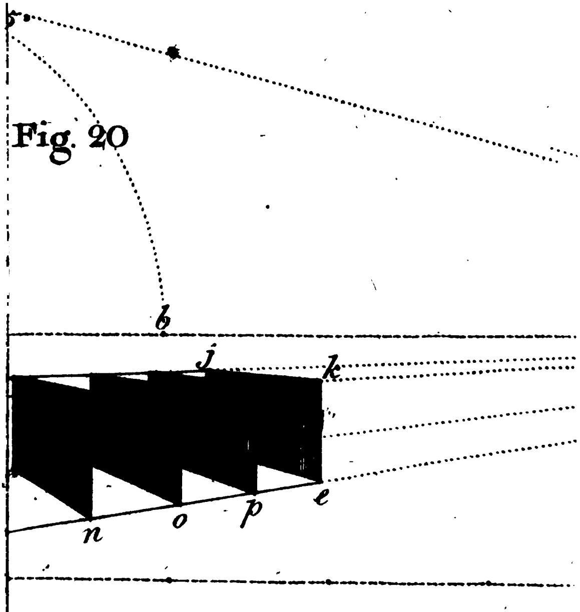

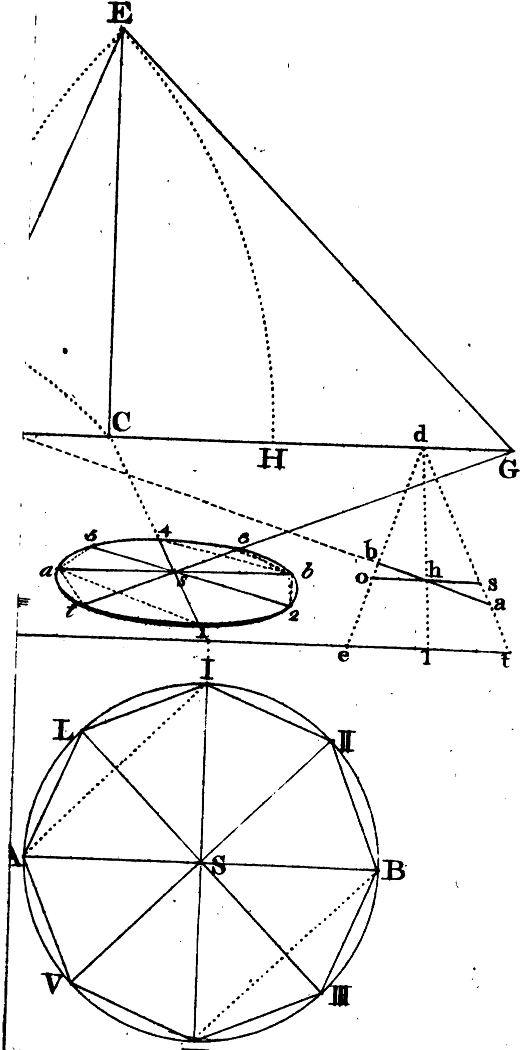
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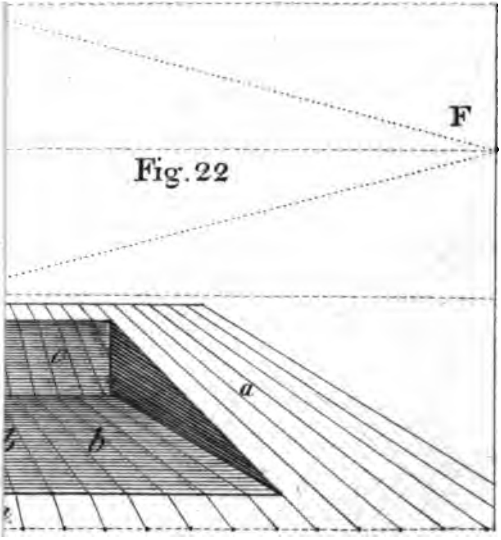
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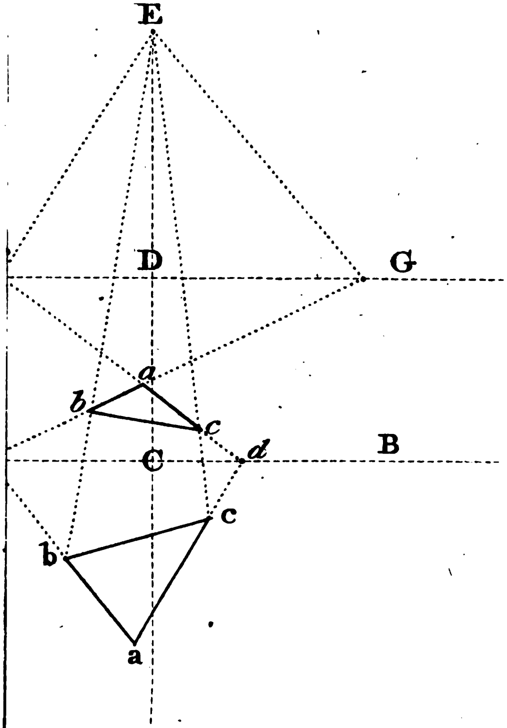

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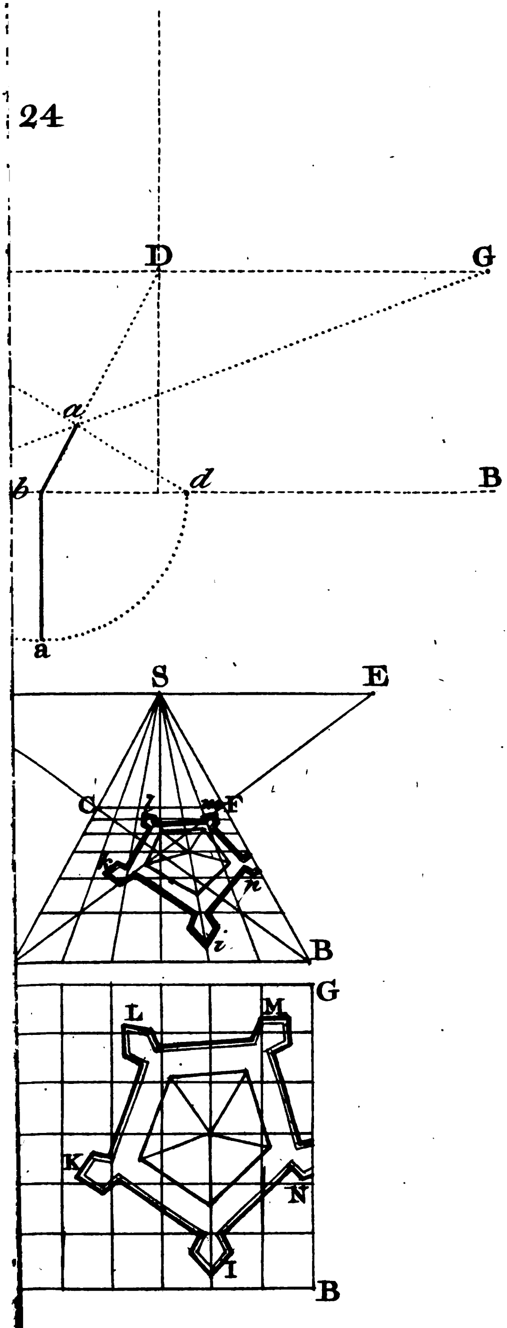

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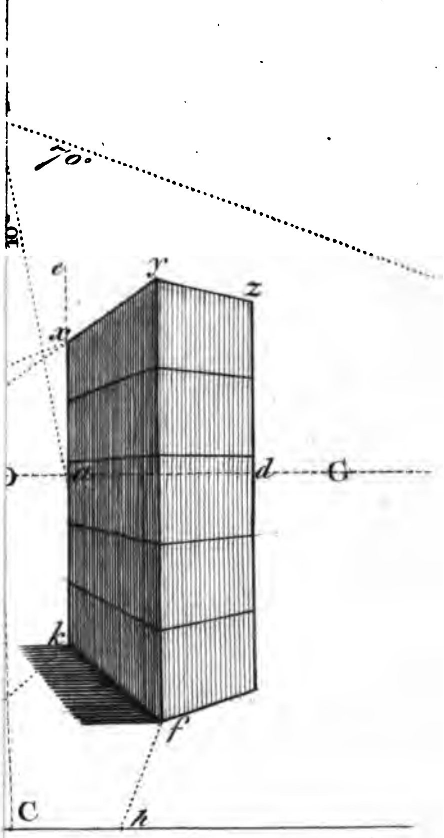

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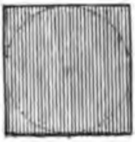

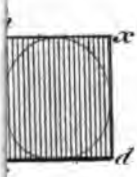

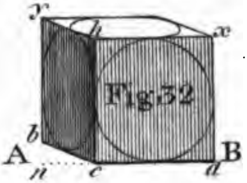

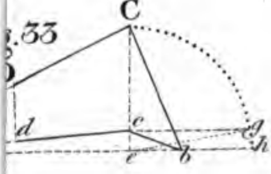

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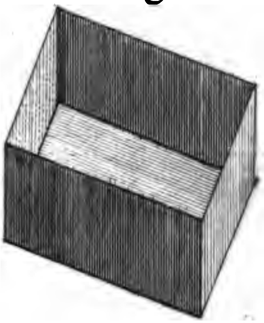

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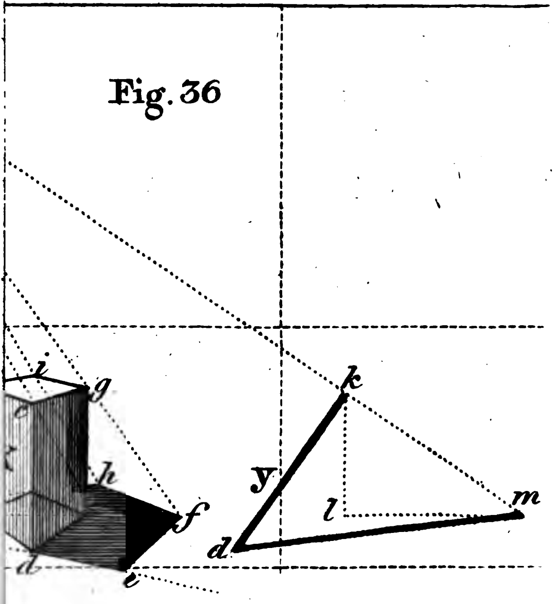

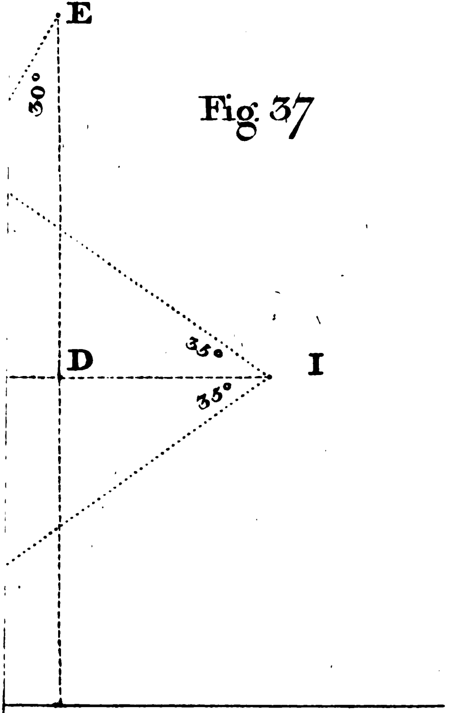

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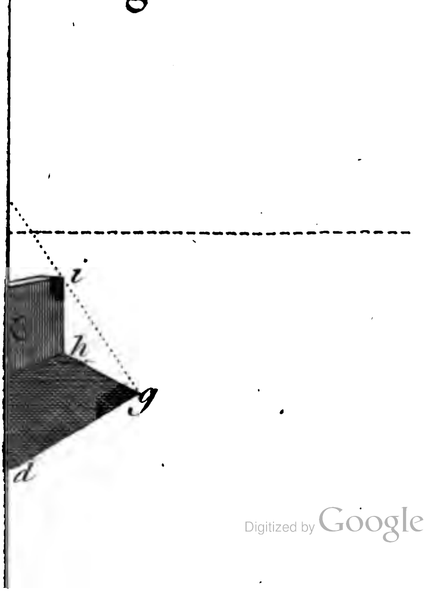

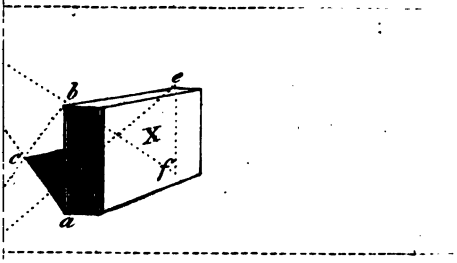

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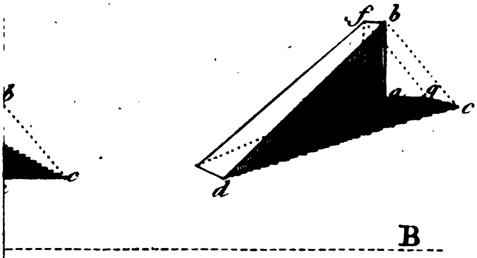

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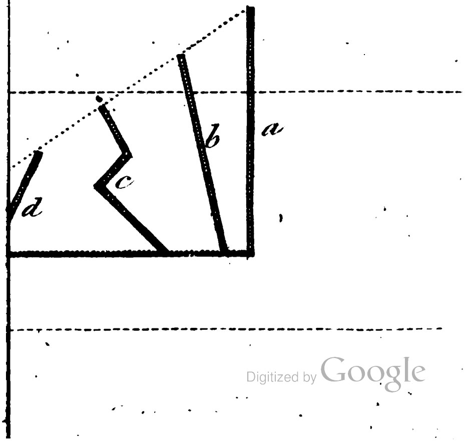

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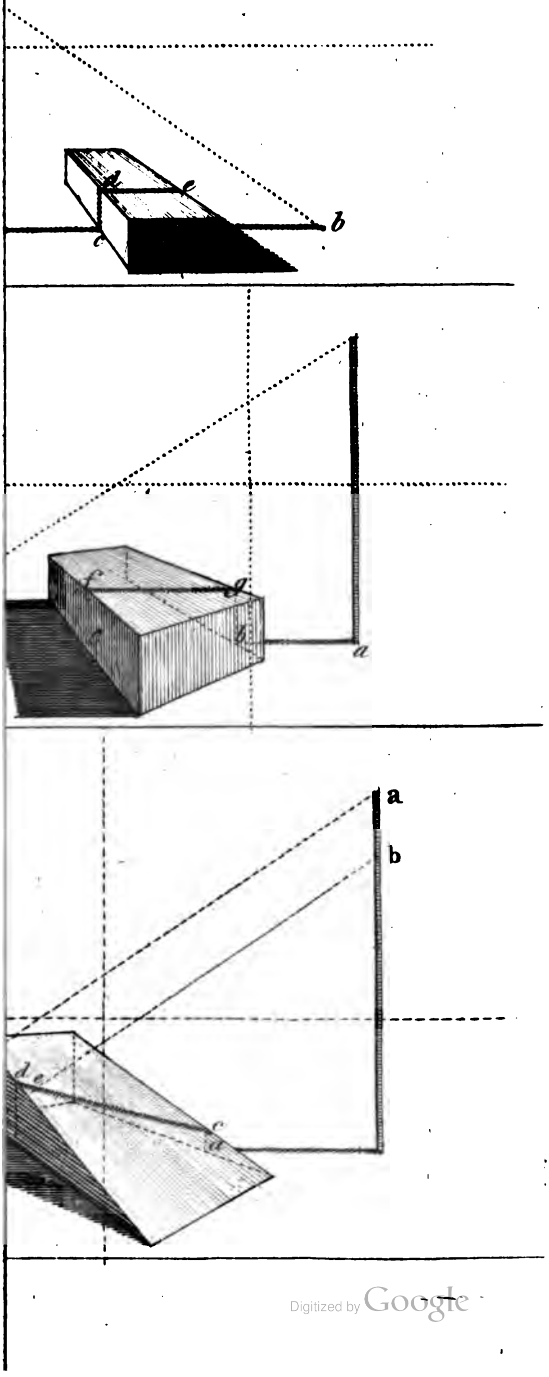

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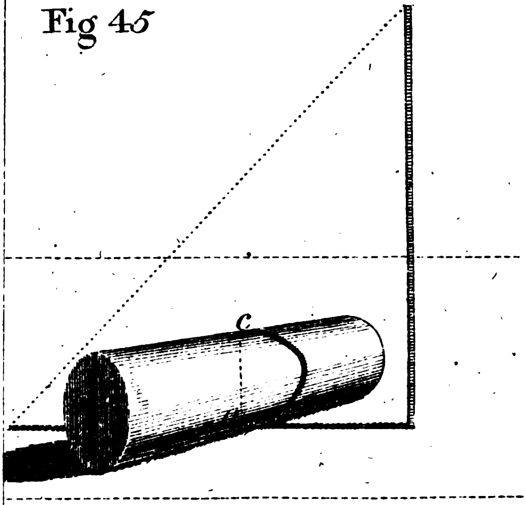

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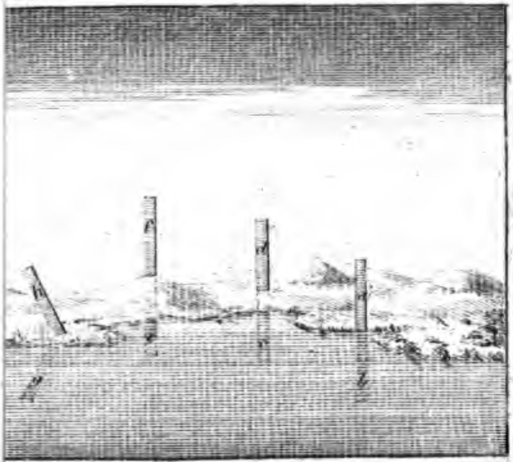

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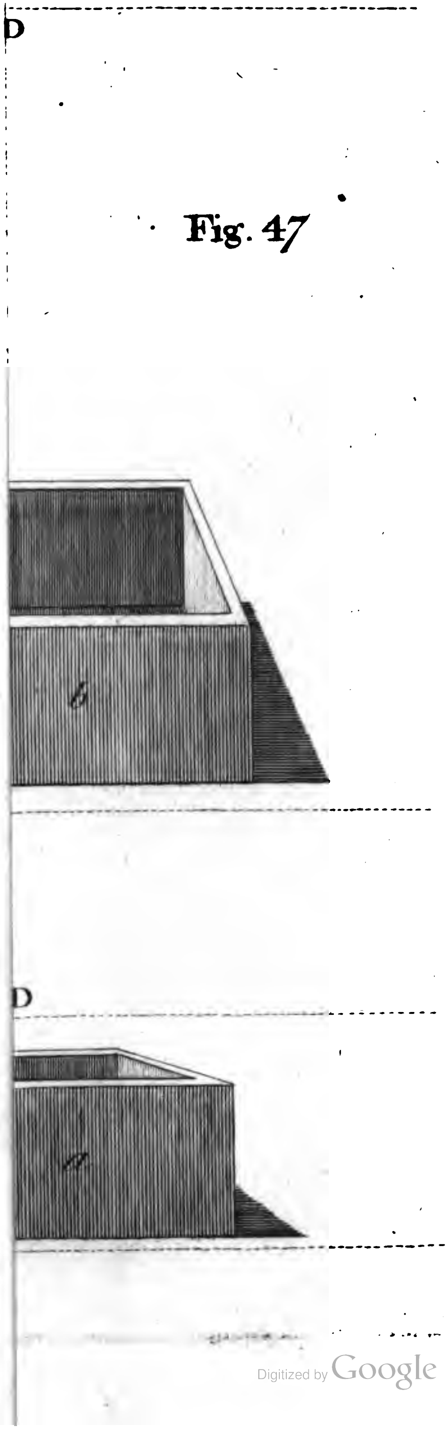

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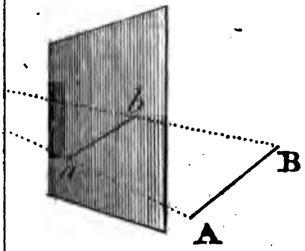

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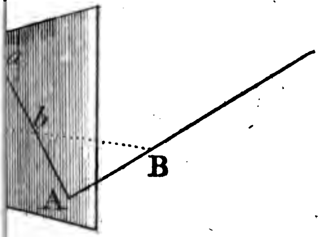

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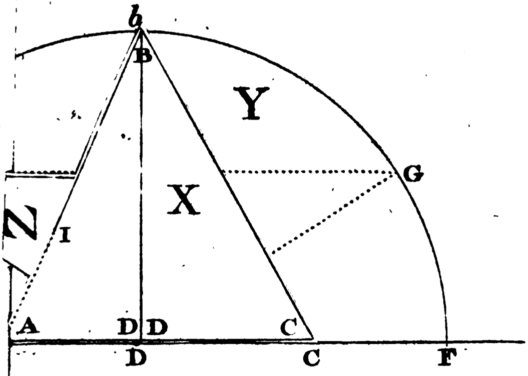

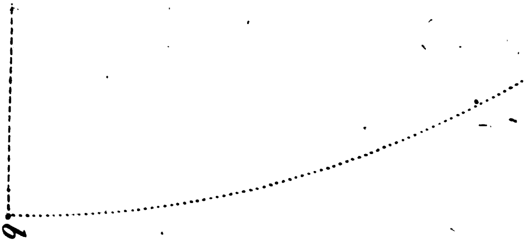

PLATE III

Hist. of Electricity

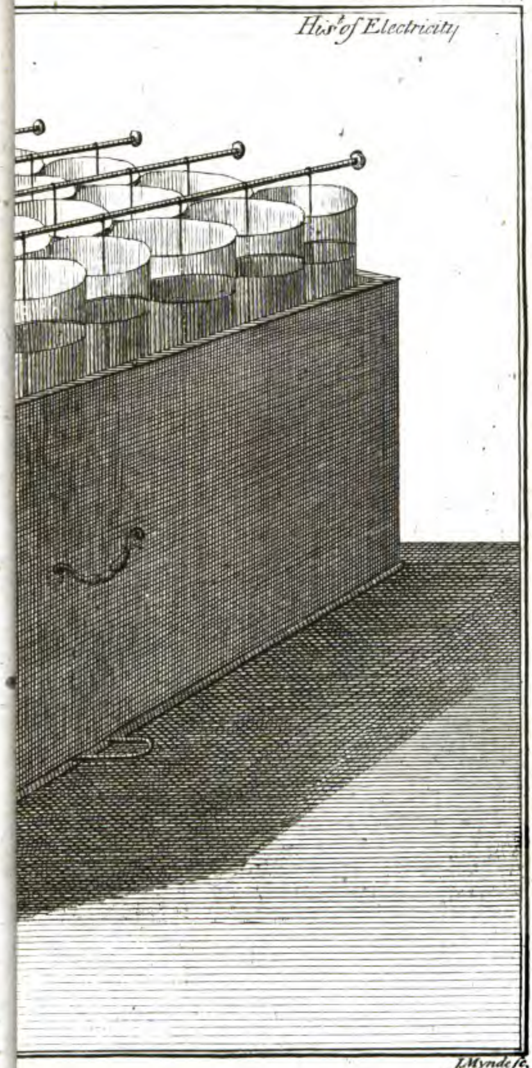

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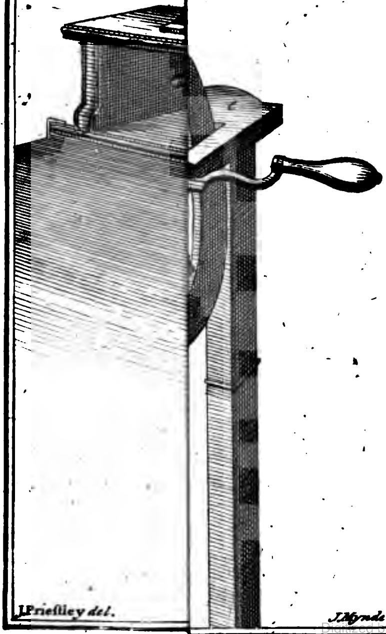
PLATE VII

Hist of Electricity



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